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Linear stability and nonlinear evolution of a polar vortex cap on a rotating sphere. (English)

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Summary: In this paper, we study the stability of a barotropic polar vortex cap on a rotating sphere. We present the linear stability analysis of the polar vortex cap, approximating the piecewise-continuous vorticity distribution by zonal bands of uniform vorticity. We investigate the dependence of the flow stability on the location of the vortex cap, modes of perturbation and rotation speed. The linear stability analysis shows that the polar vortex cap is always stable when the angular speed of the rotating body and the vorticity constant of the north are of the same sign, but is unstable when they are of different signs, regardless of the location of the cap boundary. We also compute the nonlinear evolution of the vortex cap on the rotating sphere for the unstable cases and compare it with the linear stability analysis. The flow away from the boundary of the vortex cap is the most unstable, which is in accordance with the eigenvector corresponding to the unstable eigenvalue. It is found that the solid body rotation has stabilizing effects on the evolution of the polar vortex cap.

MSC:

76E07 Rotation in hydrodynamic stability
76E20 Stability and instability of geophysical and astrophysical flows
76U05 General theory of rotating fluids
76U60 Geophysical flows

Keywords:

barotropic polar vortex; rotating sphere; vortex patch; contour dynamics; linear stability analysis

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