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Staggered explicit-implicit time-discretization for elastodynamics with dissipative internal variables. (English) Zbl 1481.65189

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Summary: An extension of the two-step staggered time discretization of linear elastodynamics in stress-velocity form to systems involving internal variables subjected to a possibly non-linear dissipative evolution is proposed. The original scheme is thus enhanced by another step for the internal variables which, in general, is implicit, although even this step might be explicit if no spatial gradients of the internal variables are involved. Using an abstract Banach-space formulation, *a priori* estimates and convergence are proved under a CFL condition. The developed three-step scheme finds applications in various problems of continuum mechanics at small strain. Here, we consider in particular plasticity, viscoelasticity (creep), diffusion in poroelastic media, and damage.

MSC:

- [65M60](#) Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
- [65M06](#) Finite difference methods for initial value and initial-boundary value problems involving PDEs
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [65M12](#) Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- [65P10](#) Numerical methods for Hamiltonian systems including symplectic integrators
- [65Z05](#) Applications to the sciences
- [74C10](#) Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)
- [74F10](#) Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)
- [74H15](#) Numerical approximation of solutions of dynamical problems in solid mechanics
- [74R20](#) Anelastic fracture and damage
- [74S05](#) Finite element methods applied to problems in solid mechanics
- [76S05](#) Flows in porous media; filtration; seepage
- [76A10](#) Viscoelastic fluids
- [26A33](#) Fractional derivatives and integrals

Keywords:

elastodynamics; explicit discretization; fractional steps; mixed finite-element method; plasticity; creep; poro-elasticity; damage

Full Text: [DOI](#) [arXiv](#)

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