Wu, Mingyuan; **Guo, Lankun**; **Li, Qingguo**

New representations of algebraic domains and algebraic L-domains via closure systems. (English) Zbl 07395089

Semigroup Forum 103, No. 2, 700-712 (2021)

Summary: Closure systems (spaces) play an important role in characterizing certain ordered structures. In this paper, *FinSet-bounded algebraic closure spaces* are introduced, and then used to provide a new approach to constructing algebraic domains. Then, a special family of algebraic closure spaces, *algebraic L-closure spaces,* are used to represent algebraic L-domains. Next, *algebraic approximate mappings* are defined and serve as the appropriate morphisms between algebraic closure spaces, respectively, algebraic L-closure spaces. On the categorical level, we show that algebraic closure spaces (respectively, algebraic L-closure spaces,) each equipped with algebraic approximate mappings as morphisms, are equivalent to algebraic domains (respectively, algebraic L-domains) with Scott continuous functions as morphisms.

MSC:

06A15 Galois correspondences, closure operators (in relation to ordered sets)

Keywords:

algebraic domain; algebraic L-domain; closure system; category equivalence

Full Text: DOI

References:

[1] Birkhoff, G., Lattice Theory (1967), Providence: American Mathematical Society, Providence · Zbl 0153.02501

[2] Caspard, N.; Monjardet, B., Some lattices of closure systems on a finite set, Discr. Math. Theor. Comput. Sci., 6, 163-190 (2004) · Zbl 1062.06005

[3] Caspard, N.; Monjardet, B., The lattices of closure systems, closure operators, and implicational systems on a finite set: a survey, Discr. Appl. Math., 127, 241-269 (2003) · Zbl 1026.06008 · doi:10.1016/S0166-218X(02)00209-3

[4] Davey, BA; Priestley, HA, Introduction to Lattices and Order (2002), Cambridge: Cambridge University Press, Cambridge· doi:10.1017/CBO9780511809088

[5] Edelman, PH, Meet-distributive lattices and the anti-exchange closure, Algebra Univ., 10, 290-299 (1980) · Zbl 0442.06004 · doi:10.1007/BF02482912

[6] Edelman, PH; Saks, ME, Combinatorial representation and convex dimension of convex geometries, Order, 5, 23-32 (1988) · Zbl 0659.06005 · doi:10.1007/BF00143895

[7] Gierz, G.; Hofmann, KH; Keimel, K.; Lawson, JD; Mislove, MW; Scott, DS, Continuous Lattices and Domains (2003), Cambridge: Cambridge University Press, Cambridge · Zbl 1088.06001 · doi:10.1017/CBO9780511542725

[8] Guo, LK; Li, QG, The categorical equivalence between algebraic domains and F-augmented closure spaces, Order, 32, 101-116 (2015) · Zbl 1337.06003 · doi:10.1007/s11083-014-9318-8

[9] Hofmann, KH; Stralka, AR, The algebraic theory of compact Lawson semilattices: applications of Galois connections to compact semilattices, Dissert. Math., 137, 1-54 (1976) · Zbl 0359.06016

[10] Larsen, K.G., Winskel, G.: Using information systems to solve domain equations effectively. In: Kahn, G., MacQueen, D.B., Plotkin, G. (eds.) Semantics of Data Types. Lecture Notes in Computer Science, vol. 173, pp. 109-129. Springer, Berlin (1984) · Zbl 0539.68019

[11] Lei, Y.; Luo, M., Rough concept lattices and domains, Ann. Pure Appl. Logic, 159, 333-340 (2009) · Zbl 1169.06004 · doi:10.1016/j.apal.2008.09.028

[12] Pfaltz, JL, Closure lattices, Discr. Math., 154, 217-236 (1996) · Zbl 0852.06002 · doi:10.1016/0012-365X(96)00043-X

[13] Pfaltz, JL; Jamison, RE, Closure systems and their structure, Inf. Sci., 139, 3-4, 275-286 (2001) · Zbl 0993.06004 · doi:10.1016/S0020-0255(01)00169-4

[14] Raney, GN, Completely distributive complete lattices, Proc. Am. Math. Soc., 3, 5, 677-680 (1952) · Zbl 0049.30304 · doi:10.1090/S0002-9939-1952-0052392-3

[15] Scott, D.: Continuous lattices. In: Lawvere, F.W. (ed.) Toposes, Algebraic Geometry and Logic. Lecture Notes in Mathematics, vol. 274, pp. 97-136. Springer, Berlin (1972)

[16] Winskel, G.: An introduction to event structures. In: de Bakker, J.W., de Roever, W.P., Rozenberg, G. (eds.) Linear Time,

Branching Time and Partial Order in Logics and Models for Concurrency. Lecture Notes in Computer Science, vol. 354, pp. 364-397. Springer, Berlin (1989)

[17] Zhao, DS, Closure spaces and completions of posets, Semigroup Forum, 90, 543-555 (2015) · Zbl 1467.06001 · doi:10.1007/s00233-015-9692-6