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Infinite series asymptotic expansions for decaying solutions of dissipative differential equations with non-smooth nonlinearity. (English) Zbl 1477.34076

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Summary: We study the precise asymptotic behavior of a non-trivial solution that converges to zero, as time tends to infinity, of dissipative systems of nonlinear ordinary differential equations. The nonlinear term of the equations may not possess a Taylor series expansion about the origin. This absence technically cripples previous proofs in establishing an asymptotic expansion, as an infinite series, for such a decaying solution. In the current paper, we overcome this limitation and obtain an infinite series asymptotic expansion, as time goes to infinity. This series expansion provides large time approximations for the solution with the errors decaying exponentially at any given rates. The main idea is to shift the center of the Taylor expansions for the nonlinear term to a non-zero point. Such a point turns out to come from the non-trivial asymptotic behavior of the solution, which we prove by a new and simple method. Our result applies to different classes of non-linear equations that have not been dealt with previously.

MSC:

34D05 Asymptotic properties of solutions to ordinary differential equations
34A25 Analytical theory of ordinary differential equations: series, transformations, transforms, operational calculus, etc.
34C05 Topological structure of integral curves, singular points, limit cycles of ordinary differential equations
34A36 Discontinuous ordinary differential equations

Full Text: DOI arXiv

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