Wei, Zhaoting  
**Twisted complexes and simplicial homotopies.** (English)  

Summary: We consider the dg-category of twisted complexes over simplicial ringed spaces. It is clear that a simplicial map \( f : (\mathcal{U}, \mathcal{R}) \to (\mathcal{V}, \mathcal{S}) \) between simplicial ringed spaces induces a dg-functor \( f^* : \text{Tw} (\mathcal{V}, \mathcal{S}) \to \text{Tw} (\mathcal{U}, \mathcal{R}) \) where \( \text{Tw} (\mathcal{U}, \mathcal{R}) \) denotes the dg-category of twisted complexes on \( (\mathcal{U}, \mathcal{R}) \). We prove that for simplicial homotopic maps \( f \) and \( g \), there exists an \( A_\infty \)-natural transformation \( \Phi : f^* \Rightarrow g^* \) between induced dg-functors. Moreover, the 0th component of \( \Phi \) is an objectwise weak equivalence. If we restrict ourselves to the full dg-subcategory of twisted perfect complexes, then we prove that \( \Phi \) admits an \( A_\infty \)-quasi-inverse when \( (\mathcal{U}, \mathcal{R}) \) satisfies some additional conditions.

**MSC:**

- 18D20 Enriched categories (over closed or monoidal categories)
- 18G55 Nonabelian homotopical algebra (MSC2010)
- 18G30 Simplicial sets; simplicial objects in a category (MSC2010)
- 14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)

**Keywords:**

- twisted complexes
- differential graded categories
- \( A_\infty \)-natural transformation
- simplicial spaces
- simplicial homotopy

**Full Text:** DOI

**References:**
