Reading, Nathan; Speyer, David E.; Thomas, Hugh

The fundamental theorem of finite semidistributive lattices. (English) Zbl 07383336

Sel. Math., New Ser. 27, No. 4, Paper No. 59, 53 p. (2021)

It is known that a finite poset L is a distributive lattice if and only if it is isomorphic to $Downset(P)$ for some finite poset P. The authors proved a similar result for semidistributive lattices. Theorem 2.1. A finite poset L is a semidistributive lattice if and only if there exists a set S with some additional structure, such that L is isomorphic to the admissible subsets of$S$ ordered by inclusion. In this case, S and its additional structure are uniquely determined by L. All of these concepts are defined in the paper. The authors study also an infinite case and prove a number of theorems on semidistributive lattices and the mentioned construction.

Reviewer: Ivan Chajda (Přerov)

MSC:

08B05 Equational logic, Mal'tsev conditions
06A15 Galois correspondences, closure operators (in relation to ordered sets)
06B15 Representation theory of lattices
06D75 Other generalizations of distributive lattices

Keywords:

distributive lattice; semidistributive lattice

Full Text: DOI arXiv

References:

[1] Adaricheva, KV; Nation, JB; Grätzer, GA; Wehrung, F., Classes of semidistributive lattices, Lattice Theory: Special Topics and Applications, 59-101 (2016), Cham: Birkhäuser, Cham · Zbl 1477.06043 · doi:10.1007/978-3-319-44236-5$_3$

[2] Adaricheva, KV; Nation, JB, Lattices of algebraic subsets and implicational classes. In: Grätzer, G.A., Wehrung, F. (eds.) Lattice Theory: Special Topics and Applications, 102-151 (2016), Cham: Birkhäuser, Cham · Zbl 1375.06005

[3] Asai, S.: Semibricks. Int. Math. Res. Not. 2020(16), 4993-5054 (2020) · Zbl 1467.16009

[4] Assem, I., Simson, D., Skowroński, A.: Elements of the Representation Theory of Associative Algebras, vol. 1. Cambridge University Press, Cambridge (2006) · Zbl 1092.16001

[5] Barbut, M., Note sur l'algèbre des techniques d'analyse hiérarchique appendix to L'analyse hiérarchique by Benjamin Matalon, 125-146 (1965), Paris: Gauthier-Villars, Paris

[6] Barnard, E., The canonical join complex, Electron. J. Combin., 26, 1, 25 (2019) · Zbl 07032096 · doi:10.37236/7866

[7] Barnard, E.; Carroll, AT; Zhu, S., Minimal inclusions of torsion classes, Algebr. Comb., 2, 5, 879-901 (2019) · Zbl 1428.05314

[8] Birkhoff, G., Rings of sets, Duke Math. J., 3, 3, 443-454 (1937) · Zbl 0017.19403 · doi:10.1215/S0012-7094-37-00334-X

[9] Birkhoff, G.: Lattice theory. Corrected reprint of the 1967 third edition. American Mathematical Society Colloquium Publications 25. American Mathematical Society, Providence (1979)

[10] Caspard, N., Santocanale, L., Wehrung, F.: Permutahedra and associahedra. In: Grätzer, G.A., Wehrung, F. (eds.) Lattice Theory: Special Topics and Applications, vol. 2, pp. 215-286. Birkhäuser, Cham (2016) · Zbl 1401.06003

[11] Day, A., Characterizations of finite lattices that are bounded-homomorphic images of sublattices of free lattices, Canad. J. Math., 31, 1, 69-78 (1979) · Zbl 0432.06007 · doi:10.4153/CJM-1979-008-x

[12] Day, A.; Nation, JB; Tschantz, S., Doubling convex sets in lattices and a generalized semidistributivity condition, Order, 6, 2, 175-180 (1989) · Zbl 0695.06005 · doi:10.1007/BF02034334

[13] Demonet, L., Iyama, O., Reading, N., Reiten, I., Thomas, H.: Lattice theory of torsion classes: Beyond tau-tilting theory (2018). arXiv:1711.01785v2

[14] Freese, R.; Ježek, J.; Nation, JB, Free Lattices (1995), Providence: American Mathematical Society, Providence· doi:10.1090/surv/042

[15] Ganter, B.; Wille, R., Formal Concept Analysis: Mathematical Foundations (1999), Berlin: Springer, Berlin · Zbl 0909.06001 · doi:10.1007/978-3-642-59830-2

[16] Garver, A.; McConville, T., Lattice properties of oriented exchange graphs and torsion classes, Algebr. Represent. Theory, 22, 43-78 (2019) · Zbl 1408.16011 · doi:10.1007/s10468-017-9757-1

[17] Gierz, G.; Hofmann, K.; Keimel, K.; Lawson, J.; Mislove, M.; Scott, D., Continuous Lattices and Domains, Encyclopedia of Mathematics and its Applications (2003), Cambridge: Cambridge University Press, Cambridge · Zbl 1088.06001 · doi:10.1017/CBO9780511542725

[18] Grätzer, GA, Lattice Theory: Foundation (2011), Basel: Springer, Basel · Zbl 1233.06001 · doi:10.1007/978-3-0348-0018-1

[19] Iyama, O.; Reading, N.; Reiten, I.; Thomas, H., Lattice structure of Weyl groups via representation theory of preprojective algebras, Compos. Math., 154, 6, 1269-1305 (2018) · Zbl 1443.16016 · doi:10.1112/S0010437X18007078

[20] Jónsson, B., Sublattices of a free lattice, Can. J. Math., 13, 256-264 (1961) · Zbl 0132.26201 · doi:10.4153/CJM-1961-021-0

[21] Markowsky, G., The factorization and representation of lattices, Trans. Am. Math. Soc., 203, 185-200 (1975) · Zbl 0302.06011 · doi:10.1090/S0002-9947-1975-0360386-3

[22] Markowsky, G., Primes, irreducibles and extremal lattices, Order, 9, 3, 265-290 (1992) · Zbl 0778.06007 · doi:10.1007/BF00383950

[23] Mizuno, Y., Classifying \(\tau \)-tilting modules over preprojective algebras of Dynkin type, Math. Z., 277, 3-4, 665-690 (2014) · Zbl 1355.16008 · doi:10.1007/s00209-013-1271-5

[24] Ore, O., Galois connexions, Trans. Am. Math. Soc., 55, 493-513 (1944) · Zbl 0060.06204 · doi:10.1090/S0002-9947-1944-0010555-7

[25] Reading, N., Lattice and order properties of the poset of regions in a hyperplane arrangement, Algebra Universalis, 50, 179-205 (2003) · Zbl 1092.06006 · doi:10.1007/s00012-003-1834-0

[26] Reading, N., Lattice congruences of the weak order, Order, 21, 4, 315-344 (2004) · Zbl 1097.20036 · doi:10.1007/s11083-005-4803-8

[27] Reading, N., Noncrossing partitions and the shard intersection order, J. Algebraic Combin., 33, 4, 483-530 (2011) · Zbl 1290.05163 · doi:10.1007/s10801-010-0255-3

[28] Reading, N., Noncrossing arc diagrams and canonical join representations, SIAM J. Discrete Math., 29, 2, 736-750 (2015) · Zbl 1314.05015 · doi:10.1137/140972391

[29] Reading, N.; Grätzer, GA; Wehrung, F., Lattice theory of the poset of regions, Lattice Theory: Special Topics and Applications, 399-487 (2016), Cham: Birkhäuser, Cham · Zbl 1404.06004 · doi:10.1007/978-3-319-44236-5$_9$

[30] Stanley, RP, Enumerative Combinatorics (2012), Cambridge: Cambridge University Press, Cambridge · Zbl 1247.05003

[31] Thomas, H.; Williams, N., Rowmotion in slow motion, Proc. Lond. Math. Soc. (3), 119, 5, 1149-1178 (2019) · Zbl 1459.06010 · doi:10.1112/plms.12251

[32] Wille, R.: Restructuring lattice theory: an approach based on hierarchies of concepts, in Ordered sets (Banff, Alta., 1981), pp. 445-470, Reidel, Boston (1982)