Edelman, Alan

The distribution and moments of the smallest eigenvalue of a random matrix of Wishart type. (English) Zbl 0738.15010

Linear Algebra Appl. 159, 55-80 (1991).

Take an $m \times n$ $(m \leq n)$ random matrix X in which each element is an independent standard normal random variable. Form the positive (semi) definite matrix $A = XX^T$. The author shows how to obtain exact expressions for the distribution and the expected value of the smallest eigenvalue of A. The author gives new results giving the distribution as a simple recursion. This includes the more difficult case when $n - m$ is an even integer, without resorting to zonal polynomials and hypergeometric functions of matrix arguments. With the recursion, one can obtain exact expressions for the density and the moments of the distribution in terms of functions usually no more complicated than polynomials, exponentials, and at worst ordinary hypergeometric functions. The author further elaborates on the special cases when $n - m = 0, 1, 2$, and 3 and gives a numerical table of the expected values for $2 \leq m \leq 25$ and $0 \leq n - m \leq 25$.

The paper contains the sections of introduction; main results; sample plots of the distributions; derivation of density formulas; expected values and other moments; $n - m = 0, 1, 2$, and 3; computation of expected values; and appendices: mathematical programs; tables of expected values; sample formulas and other uses.

Reviewer: Y.Kuo (Knoxville)

MSC:

15B52 Random matrices (algebraic aspects)
15A42 Inequalities involving eigenvalues and eigenvectors
15A18 Eigenvalues, singular values, and eigenvectors

Keywords:

random rectangular matrix; Wishart distribution; smallest eigenvalue; hypergeometric functions; expected values; moments

Software:

Mathematica

Full Text: DOI

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