Gutyanskiĭ, V.; Nesmelova, O.; Ryazanov, V.; Yefimushkin, A.
Logarithmic potential and generalized analytic functions. (English. Ukrainian original)
Zbl 07378021

Summary: The study of the Dirichlet problem in the unit disk \( D \) with arbitrary measurable data for harmonic functions is due to the famous dissertation of N. N. Luzin [“Integral and trigonometric series” (Diss.), Moskou (1915)]. Later on, the known monograph of I. N. Vekua [Generalized analytic functions. Oxford-London-New York-Paris: Pergamon Press (1962; Zbl 0100.07603)] has been devoted to boundary-value problems (only with Hölder continuous data) for the generalized analytic functions, i.e., continuous complex valued functions \( h(z) \) of the complex variable \( z = x + iy \) with generalized first partial derivatives by Sobolev satisfying equations of the form \( \partial_z h + ah + b\overline{h} = c \), where it was assumed that the complex valued functions \( a, b \) and \( c \) belong to the class \( L^p \) with some \( p > 2 \) in smooth enough domains \( D \) in \( \mathbb{C} \).

The present paper is a natural continuation of our previous articles on the Riemann, Hilbert, Dirichlet, Poincaré and, in particular, Neumann boundary-value problems for quasiconformal, analytic, harmonic, and the so-called A-harmonic functions with boundary data that are measurable with respect to logarithmic capacity. Here, we extend the corresponding results to the generalized analytic functions \( h : D \to \mathbb{C} \) with the sources \( g : \partial_z h = g \in L^p, p > 2 \), and to generalized harmonic functions \( U \) with sources \( G : \Delta U = G \in L^p, p > 2 \).

This paper contains various theorems on the existence of nonclassical solutions of the Riemann and Hilbert boundary-value problems with arbitrary measurable (with respect to logarithmic capacity) data for generalized analytic functions with sources. Our approach is based on the geometric (theoretic-functional) interpretation of boundary-values in comparison with the classical operator approach in PDE. On this basis, it is established the corresponding existence theorems for the Poincaré problem on directional derivatives and, in particular, for the Neumann problem to the Poisson equations \( \Delta U = G \) with arbitrary boundary data that are measurable with respect to logarithmic capacity. These results can be also applied to semilinear equations of mathematical physics in anisotropic and inhomogeneous media.

MSC:
30G20 Generalizations of Bers and Vekua type (pseudoanalytic, \( p \)-analytic, etc.)
30E25 Boundary value problems in the complex plane
31A25 Boundary value and inverse problems for harmonic functions in two dimensions

Keywords:
generalized analytic functions; boundary-value problems

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