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Balanced viscosity solutions to a rate-independent coupled elasto-plastic damage system.

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In nonlinear elasticity, rate-independent systems are idealized models where internal oscillations and viscous dissipations are neglected, since the (slower) scale of external loadings is dominant. On the other hand, in the latter time scale the system presents time discontinuities, corresponding to fast transitions between equilibria. In such transitions, a major role is played by the viscous dissipations.

A well-known method to study e.g. damage models is to consider a system where the flow rule for the damage variable is viscously regularized; next, one passes to the limit as the viscosity parameter tends to zero. The time discontinuities of the resulting evolution can be interpolated by means of transitions governed again by viscosity.

In this paper the authors study a model for damage coupled with plasticity, affected by viscosity both in the damage evolution and in the elastoplastic evolution. Moreover, a further dissipation source may come from a hardening process. Viscosity and hardening provide regularizing terms in the PDE system.

In the rate-independent idealization, one would neglect both viscosity and hardening. To rigorously see this, the authors consider a singularly perturbed PDE system where the terms related to viscosity and hardening are modulated by small parameters tending to zero. By tuning the speed of the convergence of such coefficients, one may model a system where the elastic and the plastic strain converge to rate-independent evolution with the same rate, or with a faster rate, than the damage variable.

Specifically, the system analyzed by the authors features three coefficients: a hardening parameter μ, a viscosity parameter ε related to damage, and a viscosity coefficient $\varepsilon\nu$ related to plasticity. In fact, ν is a rate parameter that modulates the rate of convergence of the damage variable with respect to the plastic strain. The authors study the convergence of the system as $\varepsilon \to 0$ while ν, μ are fixed, or as $\varepsilon, \nu \to 0$, or as all parameters ε, ν, μ converge to zero. In the limit, they obtain different notions of solutions, showing in the time discontinuities a single-rate or a multi-rate character. Studying various notions of rate-independent solutions is important in order to understand which of them captures the behavior of the system for small viscosities.

Reviewer: Giuliano Lazzaroni (Firenze)

MSC:

35D40 Viscosity solutions to PDEs
35B25 Singular perturbations in context of PDEs
35A15 Variational methods applied to PDEs
34A60 Ordinary differential inclusions
35Q74 PDEs in connection with mechanics of deformable solids
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

Keywords:

rate-independent systems; variational models; vanishing viscosity; BV solutions; damage; elasto-plasticity

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