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**On the maximization problem for solutions of reaction-diffusion equations with respect to their initial data.** (English) Zbl 07372601

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Summary: We consider in this paper the maximization problem for the quantity  $\int_{\Omega} u(t, x) dx$  with respect to  $u_0 =: u(0, \cdot)$ , where  $u$  is the solution of a given reaction diffusion equation. This problem is motivated by biological conservation questions. We show the existence of a maximizer and derive optimality conditions through an adjoint problem. We have to face regularity issues since non-smooth initial data could give a better result than smooth ones. We then derive an algorithm enabling to approximate the maximizer and discuss some open problems.

**MSC:**

- 35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs Cited in 1 Document
- 35B45 A priori estimates in context of PDEs
- 35B65 Smoothness and regularity of solutions to PDEs
- 35K15 Initial value problems for second-order parabolic equations
- 35K57 Reaction-diffusion equations
- 35Q92 PDEs in connection with biology, chemistry and other natural sciences
- 35Q93 PDEs in connection with control and optimization
- 92D25 Population dynamics (general)
- 92D30 Epidemiology

**Keywords:**

reaction-diffusion; control; conservation biology; optimization

**Full Text:** [DOI](#)

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