

Nguyen, K.; Amores, Víctor Jesús; Sanz, Miguel A.; Montáns, Francisco J.

Thermodynamically consistent nonlinear viscoplastic formulation with exact solution for the linear case and well-conditioned recovery of the inviscid one. (English) Zbl 1476.74013
Comput. Mech. 67, No. 5, 1349-1373 (2021).

This paper describes the development and application of a viscoplastic formulation (model) which is thermodynamically inspired and utilizes the continuum elastic corrector rate concept. The formulation circumvents the limitations of previous formulations but incorporates their beneficial attributes such as nonlinear viscosities and hardening. There are several other important characteristics of the formulation such as employing viscoplastic strain rate to distinguish between the conservative and dissipative behaviors for reverse loading, using an implicit integration algorithm, and exhibiting inviscid plasticity, viscoelasticity, and viscoplasticity as special cases. It is interesting to note that the formulation includes a well-conditioned recovery of the inviscid solution by simply setting the viscosity coefficient as zero. The paper presents a comprehensive description of the model via several sections titled as: Derivation of the model from thermodynamic principles, Incremental theory of J₂-viscoplasticity with linear isotropic hardening, Comparison with classical models, Uniaxial numerical comparisons with classical models for linear viscoplasticity, General discrete formulation: A simple backward-Euler integration algorithm for non-constant material parameters, Numerical examples, and finally conclusions. Each section contains several subsections that facilitate for the reader to follow the development of the model in a lucid manner. The authors do well to include an extensive list of references for collateral reading for an interested reader. The future extension of the model, currently developed for small strain case to the large strain (logarithmic strain) case is also indicated.

The reviewer believes that this interesting paper will be useful to the researchers working in the viscoplasticity and related areas.

Reviewer: [Vinod K. Arya \(Dallas\)](#)

MSC:

- 74C10 Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)
- 74A15 Thermodynamics in solid mechanics
- 74S99 Numerical and other methods in solid mechanics

Keywords:

viscoplastic strain rate; Perzyna model; Duvaut-Lions model; backward Euler integration algorithm

Software:

[HYPLAS](#)

Full Text: [DOI](#) [arXiv](#)

References:

- [1] Bathe, KJ, *Finite element procedures* (2014), Cambridge: Klaus-Jürgen Bathe, Cambridge · [Zbl 1326.65002](#)
- [2] Caggiano, A.; Martinelli, E.; Said Schicchi, D.; Etse, G., A modified Duvaut-Lions zero-thickness interface model for simulating the rate-dependent bond behavior of FRP-concrete joints, *Compos Part B Eng*, 149, April, 260-267 (2018) · [doi:10.1016/j.compositesb.2018.05.010](#)
- [3] Carosio, A.; Willam, K.; Etse, G., On the consistency of viscoplastic formulations, *Int J Solids Struct*, 37, 48, 7349-7369 (2000) · [Zbl 1056.74512](#) · [doi:10.1016/S0020-7683\(00\)00202-X](#)
- [4] Chaboche, JL, Constitutive equations for cyclic plasticity and cyclic viscoplasticity, *Int J Plasticity*, 5, May, 247-302 (1989) · [Zbl 0695.73001](#) · [doi:10.1016/0749-6419\(89\)90015-6](#)
- [5] Cormeau, I., Numerical stability in quasi-static elasto/visco-plasticity, *Int J Numer Methods Eng*, 9, 1, 109-127 (1975) · [Zbl 0293.73022](#) · [doi:10.1002/nme.1620090110](#)
- [6] Duvaut, G.; Lions, J., *Les Inequations en Mecanique et en Physique* (1972), Paris: Dunod, Paris · [Zbl 0298.73001](#)
- [7] Dvorkin, E.; Goldschmit, M., *Nonlinear Continua* (2005), Berlin: Springer, Berlin · [Zbl 1097.74001](#) · [doi:10.1007/3-540-29264-0](#)

- [8] Heeres, OM; Suiker, ASJ; De Borst, R., A comparison between the Perzyna viscoplastic model and the consistency viscoplastic model, *Euro J Mech A/Solids*, 21, 1, 1-12 (2002) · Zbl 1005.74013 · doi:10.1016/S0997-7538(01)01188-3
- [9] Hughes, TJ; Taylor, RL, Unconditionally stable algorithms for quasi-static elasto/visco-plastic finite element analysis, *Comput Struct*, 8, 2, 169-173 (1978) · Zbl 0365.73029 · doi:10.1016/0045-7949(78)90019-6
- [10] Ibrahimbegović, A.; Chorfi, L., Viscoplasticity model at finite deformations with combined isotropic and kinematic hardening, *Comput Struct*, 77, 5, 509-525 (2000) · doi:10.1016/S0045-7949(99)00232-1
- [11] Kowalczyk-Gajewska, K.; Pieczyska, EA; Golasinski, K.; Maj, M.; Kuramoto, S.; Furutab, T., A finite strain elastic-viscoplastic model of Gum Metal, *Int J Plasticity*, 119, 2018, 85-101 (2019) · doi:10.1016/j.ijplas.2019.02.017
- [12] Latorre, M.; Montáns, F., Anisotropic finite strain viscoelasticity based on the Sidoroff multiplicative decomposition and logarithmic strains, *Comput Mech*, 56, 503-531 (2016) · Zbl 1326.74031 · doi:10.1007/s00466-015-1184-8
- [13] Latorre, M.; Montáns, F., Fully anisotropic finite strain viscoelasticity based on a reverse multiplicative decomposition and logarithmic strains, *Comput Struct*, 163, 56-70 (2016) · doi:10.1016/j.compstruc.2015.09.001
- [14] Latorre, M.; Montáns, F., A new class of plastic flow evolution equations for anisotropic multiplicative elastoplasticity based on the notion of a corrector elastic strain rate, *Appl Math Modell*, 55, 716-740 (2018) · Zbl 1480.74031 · doi:10.1016/j.apm.2017.11.003
- [15] Latorre, M.; Montáns, F., Bi-modulus materials consistent with a stored energy function: Theory and numerical implementation, *Comput Struct*, 229, 106176 (2020) · doi:10.1016/j.compstruc.2019.106176
- [16] Lubliner, J., *Plasticity Theory* (1990), Cambridge: Macmillan, Cambridge · Zbl 0745.73006
- [17] Kojic M, Bathe K, (2005) *Inelastic analysis of solids and structures*. Springer, Berlin
- [18] Miehe, C.; Schröder, J., A comparative study of stress update algorithms for rate-independent and rate-dependent crystal plasticity, *Int J Numer Methods Eng*, 50, 273-298 (2001) · Zbl 1006.74091 · doi:10.1002/1097-0207(20010120)50:2<273::AID-NME17>3.0.CO;2-Q
- [19] Nedjar, B., Frameworks for finite strain viscoelastic-plasticity based on multiplicative decompositions. Part I: Continuum formulations, *Comput Methods Appl Mech Eng*, 191, 15-16, 1541-1562 (2002) · Zbl 1141.74317 · doi:10.1016/S0045-7825(01)00337-1
- [20] Nguyen K, Sanz M, Montáns F (2020) Plane-stress constrained multiplicative hyperelasto-plasticity with nonlinear kinematic hardening, consistent theory based on elastic corrector rates and algorithmic implementation. *International Journal of Plasticity* 128:102592
- [21] Peric, D., On a class of constitutive equations in viscoplasticity: formulation and computational issues, *Int J Numer Methods Eng*, 36, 8, 1365-1393 (1993) · Zbl 0815.73020 · doi:10.1002/nme.1620360807
- [22] Perić, D., On a class of constitutive equations in viscoplasticity: formulation and computational issues, *Int J Numer Methods Eng*, 36, 1365-1393 (1993) · Zbl 0815.73020 · doi:10.1002/nme.1620360807
- [23] Perzyna, P., *Fundamental Problems in Viscoplasticity*, *Adv Appl Mech*, 9, C, 243-377 (1966) · doi:10.1016/S0065-2156(08)70009-7
- [24] Ristinmaa, M.; Ottosen, NS, Viscoplasticity based on an additive split of the conjugated forces, *Eur J Mech A/Solids*, 17, 2, 207-235 (1998) · Zbl 0924.73075 · doi:10.1016/S0997-7538(98)80083-1
- [25] Ristinmaa, M.; Ottosen, NS, Consequences of dynamic yield surface in viscoplasticity, *Int J Solids Struct*, 37, 33, 4601-4622 (2000) · Zbl 0982.74014 · doi:10.1016/S0020-7683(99)00158-4
- [26] Runesson K, Ristinmaa M, Mahler L (1999) Comparison of viscoplasticity formats and algorithms. *Mechanics of Cohesive-Frictional Materials* 4(1):75-98
- [27] Sanz, M.; Nguyen, K.; Latorre, M.; Rodríguez, M.; Montáns, F., Sheet metal forming analysis using a large strain anisotropic multiplicative plasticity formulation, based on elastic correctors, which preserves the structure of the infinitesimal theory, *Finite Elements Anal Design*, 164, 1-17 (2019) · doi:10.1016/j.finel.2019.06.004
- [28] Sanz, MA; Montáns, F.; Latorre, M., Computational anisotropic hardening multiplicative elastoplasticity based on the corrector elastic logarithmic strain rate, *Comput Methods Appl Mech Eng*, 320, 82-121 (2017) · Zbl 1439.74065 · doi:10.1016/j.cma.2017.02.027
- [29] Shutov, AV; Kreißig, R., Finite strain viscoplasticity with nonlinear kinematic hardening: Phenomenological modeling and time integration, *Comput Methods Appl Mech Eng*, 197, 21-24, 2015-2029 (2008) · Zbl 1194.74026 · doi:10.1016/j.cma.2007.12.017
- [30] Simo, JC; Hughes, TJR, *Computational inelasticity* (1998), Berlin: Springer, Berlin · Zbl 0934.74003
- [31] Simo, JC; Kennedy, JG; Govindjee, S., Non-Smooth Multisurface Plasticity and Viscoplasticity. Loading / Unloading Conditions and Numerical Algorithms, *Int J Numer Methods Eng*, 26, 1987, 2161-2185 (1988) · Zbl 0661.73058 · doi:10.1002/nme.1620261003
- [32] de Souza-Neto, EA; Peric, D.; Owen, D., *Computational methods for plasticity: theory and applications* (2008), New York: Wiley, New York · doi:10.1002/9780470694626
- [33] Truesdell, C.; Noll, W., *The non-linear field theories of mechanics* (2004), Berlin: Springer, Berlin · Zbl 0779.73004 · doi:10.1007/978-3-662-10388-3
- [34] Wang, H.; Wu, P.; Tomé, C.; Huang, Y., A finite strain elastic-viscoplastic self-consistent model for polycrystalline materials, *J Mech Phys Solids*, 58, 4, 594-612 (2010) · Zbl 1244.74036 · doi:10.1016/j.jmps.2010.01.004
- [35] Wang WM, Shys LJ, Borst dRR (1997) Viscoplasticity for instabilities due to strain softening and strain-rate softening. *International Journal for Numerical Methods in Engineering* 40(20), 3839-3864 · Zbl 0974.74511
- [36] Wilkins ML (1963) Calculation of elastic-plastic flow. Tech. Rep. UCRL-7322, University of California, Lawrence Radiation Laboratory, Livermore
- [37] Zaera, R.; Fernández-Sáez, J., An implicit consistent algorithm for the integration of thermoviscoplastic constitutive equations

in adiabatic conditions and finite deformations, *Int J Solids Struct*, 43, 6, 1594-1612 (2006) · [Zbl 1120.74838](#) · [doi:10.1016/j.ijsolstr.2005.03.070](#)

- [38] Zhang, M.; Montáns, F., A simple formulation for large-strain cyclic hyperelasto-plasticity using elastic correctors. theory and algorithmic implementation, *Int J Plasticity*, 113, 185-217 (2019) · [doi:10.1016/j.ijplas.2018.09.013](#)
- [39] Zienkiewicz, OC; Corneau, IC, Visco-Plasticity-Plasticity and Creep in Elastic Solids—A unified numerical solution approach, *Int J Numer Methods Eng*, 8, March, 821-845 (1974) · [Zbl 0284.73021](#) · [doi:10.1002/nme.1620080411](#)

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