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Stabilization of the response of cyclically loaded lattice spring models with plasticity. (English) Zbl 07369299

ESAIM, Control Optim. Calc. Var. 27, Suppl., Paper No. S8, 43 p. (2021)

Summary: This paper develops an analytic framework to design both stress-controlled and displacement-controlled T-periodic loadings which make the quasistatic evolution of a one-dimensional network of elastoplastic springs converging to a unique periodic regime. The solution of such an evolution problem is a function $t \mapsto (e(t), p(t))$, where $e_i(t)$ is the elastic elongation and $p_i(t)$ is the relaxed length of spring i, defined on $[t_0, \infty)$ by the initial condition $(e(t_0), p(t_0))$. After we rigorously convert the problem into a Moreau sweeping process with a moving polyhedron $C(t)$ in a vector space E of dimension d, it becomes natural to expect (based on a result by *P. Krejčí* [Hysteresis, convexity and dissipation in hyperbolic equations. Tokyo: Gakkotosho (1996; Zbl 1187.35003)]) that the elastic component $t \mapsto e(t)$ always converges to a T-periodic function as $t \to \infty$. The achievement of this paper is in spotting a class of loadings where the Krejci's limit does not depend on the initial condition $(e(t_0), p(t_0))$ and so all the trajectories approach the same T-periodic regime. The proposed class of sweeping processes is the one for which the normals of any d different facets of the moving polyhedron $C(t)$ are linearly independent. We further link this geometric condition to mechanical properties of the given network of springs. We discover that the normals of any d different facets of the moving polyhedron $C(t)$ are linearly independent, if the number of displacement-controlled loadings is two less the number of nodes of the given network of springs and when the magnitude of the stress-controlled loading is sufficiently large (but admissible). The result can be viewed as an analogue of the high-gain control method for elastoplastic systems. In continuum theory of plasticity, the respective result is known as Frederick-Armstrong theorem.

MSC:

74H55 Stability of dynamical problems in solid mechanics
74M05 Control, switches and devices ("smart materials") in solid mechanics
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
93C15 Control/observation systems governed by ordinary differential equations

Keywords:

elastoplasticity; Moreau sweeping process; quasistatic evolution; periodic loading; stress control; displacement control; stabilization

Full Text: DOI arXiv

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