

Koutras, Costas D.; Pitsiladis, Georgios V.

Galois connections for bilattices. (English) Zbl 07364297
Algebra Univers. 82, No. 3, Paper No. 37, 33 p. (2021)

Summary: We introduce notions of *Galois biconnections*, intended to be the bilattice analogue of classical Galois connections between lattices. A *bidirectional Galois biconnection* is a (compatible) pair of Galois connections, the first relating *truth orderings* and the second relating *knowledge orderings*, while a *unidirectional Galois biconnection* is a Galois connection equipped with extra properties that seek to capture the bilattice structure. A further distinction is made between *strong Galois biconnections* which furnish bilattice-isomorphic images and *regular Galois biconnections* which induce order-isomorphic images of the maps. We investigate all four species of Galois biconnections on pre-bilattices and on bilattices with negation and conflation. We examine both the survival of elegant properties of Galois connections (composability, invertibility, preservation of joins and meets, etc.) and the preservation of interesting bilattice properties (distributivity, boundedness, interlacing) for the images of the bilattices under the Galois biconnection. Finally, we discuss the naturally emerging biclosure operators on bilattices.

MSC:

- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 06B05 Structure theory of lattices
- 03G25 Other algebras related to logic

Keywords:

Galois connections; bilattices; closure operators

Full Text: [DOI](#)

References:

- [1] Arieli, O., Paraconsistent declarative semantics for extended logic programs, *Ann. Math. Artif. Intell.*, 36, 381-417 (2002) · [Zbl 1015.68121](#) · [doi:10.1023/A:1016358201013](#)
- [2] Arieli, O., Reasoning with different levels of uncertainty, *J. Appl. Non-Class. Log.*, 13, 317-343 (2003) · [Zbl 1180.68268](#) · [doi:10.3166/jancl.13.317-343](#)
- [3] Arieli, O.: A tutorial on bilattices (2012). Tutorial in Duality Theory in Algebra, Logic and Computer Science, Oxford, UK, June 13-14, 2012
- [4] Arieli, O.; Avron, A., Reasoning with logical bilattices, *J. Log. Lang. Inf.*, 5, 25-63 (1996) · [Zbl 0851.03017](#) · [doi:10.1007/BF00215626](#)
- [5] Arieli, O.; Denecker, M., Reducing preferential paraconsistent reasoning to classical entailment, *J. Logic Comput.*, 13, 557-580 (2003) · [Zbl 1034.03023](#) · [doi:10.1093/logcom/13.4.557](#)
- [6] Bergman, C., *Universal Algebra: Fundamentals and Selected Topics* (2011), Boca Raton: Chapman and Hall/CRC, Boca Raton · [doi:10.1201/9781439851302](#)
- [7] Birkhoff, G., *Lattice Theory* (1940), New York: American Mathematical Society Colloquium Publications, New York · [Zbl 0063.00402](#)
- [8] Bou, F.; Jansana, R.; Rivieccio, U., Varieties of interlaced bilattices, *Algebra Universalis*, 66, 115-141 (2011) · [Zbl 1231.06009](#) · [doi:10.1007/s00012-011-0151-2](#)
- [9] Cabrer, LM; Craig, APK; Priestley, HA, Product representation for default bilattices: an application of natural duality theory, *J. Pure Appl. Algebra*, 219, 2962-2988 (2015) · [Zbl 1311.06009](#) · [doi:10.1016/j.jpaa.2014.09.034](#)
- [10] Cabrer, LM; Priestley, HA, A general framework for product representations: bilattices and beyond, *Log. J. IGPL*, 23, 816-841 (2015) · [Zbl 1405.06002](#) · [doi:10.1093/jigpal/jzv030](#)
- [11] Cabrer, LM; Priestley, HA, Natural dualities through product representations: bilattices and beyond, *Studia Logica*, 104, 567-592 (2016) · [Zbl 1360.08005](#) · [doi:10.1007/s11225-016-9651-6](#)
- [12] Davey, BA; Priestley, HA, *Introduction to Lattices and Order* (2002), Cambridge: Cambridge University Press, Cambridge · [doi:10.1017/CBO9780511809088](#)
- [13] Denecke, K.; Ern , M.; Wismath, SL, *Galois Connections and Applications* (2004), Berlin: Springer, Berlin · [Zbl 1050.06001](#)
- [14] Ern , M.: Adjunctions and Galois Connections: Origins, History and Development, pp. 1-138. In: Denecke et al. [13] (2004) · [Zbl 1067.06003](#)

- [15] Ern , M.; Koslowski, J.; Melton, A.; Strecker, GE, A primer on Galois connections, *Ann. NY Acad. Sci.*, 704, 103-125 (1993) · [Zbl 0809.06006](#) · [doi:10.1111/j.1749-6632.1993.tb52513.x](#)
- [16] Fitting, M., Bilattices and the theory of truth, *J. Philos. Logic*, 18, 225-256 (1989) · [Zbl 0678.03028](#) · [doi:10.1007/BF00274066](#)
- [17] Fitting, M., Bilattices and the semantics of logic programming, *J. Logic Progr.*, 11, 91-116 (1991) · [Zbl 0757.68028](#) · [doi:10.1016/0743-1066\(91\)90014-G](#)
- [18] Fitting, M., Kleene’s logic, generalized, *J. Logic Comput.*, 1, 797-810 (1991) · [Zbl 0744.03025](#) · [doi:10.1093/logcom/1.6.797](#)
- [19] Fitting, M., The family of stable models, *J. Logic Progr.*, 17, 197-225 (1993) · [Zbl 0798.68096](#) · [doi:10.1016/0743-1066\(93\)90031-B](#)
- [20] Fitting, M., Kleene’s three valued logics and their children, *Fund. Inform.*, 20, 113-131 (1994) · [Zbl 0804.03016](#)
- [21] Fitting, M.; Bolander, T.; Hendricks, V.; Pedersen, SA, Bilattices are nice things, *Self-Reference* (2006), Stanford: CSLI Publications, Stanford · [Zbl 1157.03308](#)
- [22] Ginsberg, ML, Multivalued logics: a uniform approach to reasoning in Artificial Intelligence, *Comput. Intell.*, 4, 265-316 (1988) · [doi:10.1111/j.1467-8640.1988.tb00280.x](#)
- [23] Jansana, R.; Rivieccio, U., Residuated bilattices, *Soft Comput.*, 16, 493-504 (2012) · [Zbl 1255.03058](#) · [doi:10.1007/s00500-011-0752-x](#)
- [24] Jung, A.; Rivieccio, U., Priestley duality for bilattices, *Studia Logica*, 100, 223-252 (2012) · [Zbl 1258.06008](#) · [doi:10.1007/s11225-012-9376-0](#)
- [25] Melton, A.; Schmidt, DA; Strecker, GE, Galois connections and computer science applications, *Category Theory and Computer Programming*, *Lecture Notes in Computer Science*, 299-312 (1986), Berlin: Springer, Berlin
- [26] Mobasher, B.; Pigozzi, D.; Slutzki, G.; Voutsadakis, G., A duality theory for bilattices, *Algebra Universalis*, 43, 109-125 (2000) · [Zbl 1012.06008](#) · [doi:10.1007/s000120050149](#)
- [27] Ore, O., Galois connexions, *Trans. Am. Math. Soc.*, 55, 493-513 (1944) · [Zbl 0060.06204](#) · [doi:10.1090/S0002-9947-1944-0010555-7](#)
- [28] Pitsiladis, G.: Notions of Galois connections for bilattices. Master’s thesis, ALMA, Graduate Program in Algorithms, Logic and Discrete Mathematics, NTUA and Athens University (2018)
- [29] Rivieccio, U.: An algebraic study of bilattice-based logics. PhD, Universitat de Barcelona (2010)
- [30] Rivieccio, U., Representation of interlaced trilattices, *J. Appl. Logics*, 11, 174-189 (2013) · [Zbl 1284.03282](#) · [doi:10.1016/j.jal.2013.03.001](#)
- [31] Rivieccio, U.; Gor , R.; Kooi, BP; Kurucz, A., Bilattice public announcement logic, *Advances in Modal Logic (AiML 10)*, 459-477 (2014), London: College Publications, London · [Zbl 1385.03008](#)
- [32] Shramko, Y.; Wansing, H., Some useful 16-valued logics: How a computer network should think, *J. Philos. Logic*, 34, 121-153 (2005) · [Zbl 1094.03012](#) · [doi:10.1007/s10992-005-0556-5](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.