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Two- and three-dimensional size-dependent couple stress response using a displacement-based variational method. (English) Zbl 07362918

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Summary: Based upon the principle of minimum total potential energy for consistent couple stress theory, a Ritz variational approach is developed using tensor product B-splines for both two- and three-dimensional elastostatic analysis. The underlying theory is size-dependent due to incorporation of the energy conjugate skew-symmetric couple-stress and mean curvature tensors, in addition to the force-stress and strain conjugate pair of classical theory. The use of B-splines as the basis functions can assure the required C^1 continuity of the displacement field, while also permitting higher order representations. Both displacements and rotations are essential boundary conditions in this theory. Displacements can be enforced in the usual way, but rotations require special treatment to maintain symmetry and positive definiteness of the stiffness matrix for well-posed elastostatic problems. Several computational examples are considered to validate the formulation, illustrate convergence characteristics, and investigate mechanical behavior under consistent couple stress theory. All previous numerical analysis of consistent couple stress theory has been limited to plane strain problems. Thus, the extension here to three-dimensions is of great importance, especially because the behavior in several cases shows significant deviation from the plane strain solutions. Consequently, the variational formulations and computational methodology presented in this paper can play a critical role in assessing the predictive capability of consistent couple stress theory and in understanding size-dependent elastic response.

MSC:

74-XX Mechanics of deformable solids

Keywords:

size-dependent mechanics; consistent couple stress theory; skew-symmetric couple-stress and mean curvature tensors; C^1 continuous displacement field; micromechanics and nanomechanics

Software:

Matlab

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