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Numerical analysis of a viscoplastic contact problem with normal compliance, unilateral constraint, memory term and friction. (English) Zbl 1476.65255

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Summary: In this paper, we consider a model which describes the quasistatic contact between a viscoplastic body and a foundation. The material's behavior is modeled with a rate-type viscoplastic constitutive law with an internal state variable. The contact is modeled with normal compliance, unilateral constraint, memory term, and friction which is under a total slip-dependent version of Coulomb's law. For the weak formulation of the problem, which is in the form of a system coupling two nonlinear integral equations with a history-dependent variational-hemivariational inequality, we introduce a fully discrete scheme and derive an error estimate. Under appropriate regularity assumptions, we obtain an optimal-order error estimate in finite element spaces. Finally, numerical results are reported to show the performance of the numerical method.

MSC:

- [65M60](#) Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
- [65M06](#) Finite difference methods for initial value and initial-boundary value problems involving PDEs
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [65M15](#) Error bounds for initial value and initial-boundary value problems involving PDEs
- [65R20](#) Numerical methods for integral equations
- [74M15](#) Contact in solid mechanics
- [74C10](#) Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)
- [74D10](#) Nonlinear constitutive equations for materials with memory
- [35Q74](#) PDEs in connection with mechanics of deformable solids

Keywords:

variational-hemivariational inequality; viscoplasticity; friction; numerical approximation; optimal-order error estimate

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