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Propagation in a fractional reaction-diffusion equation in a periodically hostile environment.

(English) [Zbl 1464.35398](#)

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Summary: We provide an asymptotic analysis of a fractional Fisher-KPP type equation in periodic non-connected media with Dirichlet conditions outside the domain. After showing the existence and uniqueness of a non-trivial bounded stationary state n_+ , we prove that it invades the unstable state zero exponentially fast in time.

MSC:

35R11 Fractional partial differential equations

35K20 Initial-boundary value problems for second-order parabolic equations

35K08 Heat kernel

35K57 Reaction-diffusion equations

35B40 Asymptotic behavior of solutions to PDEs

35Q92 PDEs in connection with biology, chemistry and other natural sciences

Keywords:

non-local fractional operator; Fisher KPP; asymptotic analysis; exponential speed of propagation; perturbed test function

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