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Hom-derivations in C^* -ternary algebras. (English) Zbl 07343745

Acta Math. Sin., Engl. Ser. 36, No. 9, 1025-1038 (2020).

Summary: In this paper, we introduce and solve the following additive (ρ_1, ρ_2) -functional inequalities

$$\begin{aligned} & \|f(x + y + z) - f(x) - f(y) - f(z)\| \\ & \leq \|\rho_1(f(x + z) - f(x) - f(z))\| + \|\rho_2(f(y + z) - f(y) - f(z))\|, \end{aligned}$$

where ρ_1 and ρ_2 are fixed nonzero complex numbers with $|\rho_1| + |\rho_2| < 2$. Using the fixed point method and the direct method, we prove the Hyers-Ulam stability of the above additive (ρ_1, ρ_2) -functional inequality in complex Banach spaces. Furthermore, we prove the Hyers-Ulam stability of hom-derivations in C^* -ternary algebras.

Reviewer: [Reviewer \(Berlin\)](#)

MSC:

- [39B62](#) Functional inequalities, including subadditivity, convexity, etc.
- [16W25](#) Derivations, actions of Lie algebras
- [39B82](#) Stability, separation, extension, and related topics for functional equations
- [47H10](#) Fixed-point theorems
- [39B52](#) Functional equations for functions with more general domains and/or ranges
- [47B47](#) Commutators, derivations, elementary operators, etc.
- [46L57](#) Derivations, dissipations and positive semigroups in C^* -algebras

Keywords:

additive (ρ_1, ρ_2) -functional inequality; fixed point method; direct method; hom-derivation on C^* -ternary algebra

Full Text: [DOI](#)

References:

- [1] Amyari, M.; Moslehian, M. S., Approximate homomorphisms of ternary semigroups, Lett. Math. Phys., 77, 1-9 (2006) · [Zbl 1112.39021](#)
- [2] Aoki, T., On the stability of the linear transformation in Banach spaces, J. Math. Soc. Japan, 2, 64-66 (1950) · [Zbl 0040.35501](#)
- [3] J. Inequal. Pure Appl. Math.200341
- [4] Cădariu, L.; Radu, V., On the stability of the Cauchy functional equation: a fixed point approach, Grazer Math. Ber., 346, 43-52 (2004) · [Zbl 1060.39028](#)
- [5] Cădariu, L., Radu, V.: Fixed point methods for the generalized stability of functional equations in a single variable. Fixed Point Theory Appl., 2008, Art. ID 749392 (2008) · [Zbl 1146.39040](#)
- [6] Diaz, J.; Margolis, B., A fixed point theorem of the alternative for contractions on a generalized complete metric space, Bull. Am. Math. Soc., 74, 305-309 (1968) · [Zbl 0157.29904](#)
- [7] Eghbali, N.; Rassias, J. M.; Taheri, M., On the stability of a k-cubic functional equation in intuitionistic fuzzy n-normed spaces, Results Math., 70, 233-248 (2016) · [Zbl 1360.39022](#)
- [8] Eskandani, G. Z.; Găvruta, P., Hyers-Ulam-Rassias stability of pexiderized Cauchy functional equation in 2-Banach spaces, J. Nonlinear Sci. Appl., 5, 459-465 (2012) · [Zbl 1297.39030](#)
- [9] Găvruta, P., A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, J. Math. Anal. Appl., 184, 431-436 (1994) · [Zbl 0818.46043](#)
- [10] Hyers, D. H., On the stability of the linear functional equation, Proc. Nat. Acad. Sci. U.S.A., 27, 222-224 (1941) · [Zbl 67.0424.01](#)
- [11] Isac, H.; Rassias, T. M., Stability of ψ -additive mappings: Applications to nonlinear analysis, Int. J. Math. Math. Sci., 19, 219-228 (1996) · [Zbl 0843.47036](#)
- [12] Ji, P. S.; Zhao, Y. Z., Jensen-quadratic functional equation and its Hyers-Ulam stability, Acta Math. Sin., Chin. Ser., 58,

251-260 (2015) · [Zbl 1340.39043](#)

- [13] Ji, P. S.; Zhou, S. J.; Xue, H. Y., On a Jensen-cubic functional equation and its Hyers-Ulam stability, *Acta Math. Sin., Engl. Ser.*, 31, 1929-1940 (2015) · [Zbl 1334.39053](#)
- [14] Khodaei, H., On the stability of additive, quadratic, cubic and quartic set-valued functional equations, *Results Math.*, 68, 1-10 (2015) · [Zbl 1330.39029](#)
- [15] Miheţ, D.; Radu, V., On the stability of the additive Cauchy functional equation in random normed spaces, *J. Math. Anal. Appl.*, 343, 567-572 (2008) · [Zbl 1139.39040](#)
- [16] Mirmostafae, A. K.; Mahdavi, M., The fixed point alternative theorem and set-valued functional equations, *Fixed Point Theory*, 15, 495-500 (2014) · [Zbl 1308.39025](#)
- [17] Mirzavaziri, M.; Moslehian, M. S., Automatic continuity of σ -derivations on C^* -algebras, *Proc. Am. Math. Soc.*, 134, 3319-3327 (2006) · [Zbl 1116.46061](#)
- [18] Mirzavaziri, M.; Moslehian, M. S., σ -derivations in Banach algebras, *Bull. Iran. Math. Soc.*, 32, 65-78 (2007) · [Zbl 1144.47032](#)
- [19] Moslehian, M. S., Almost derivations on C^* -ternary rings, *Bull. Belg. Math. Soc.-Simon Stevin*, 14, 135-142 (2006) · [Zbl 1132.39026](#)
- [20] Park, C., Homomorphisms between Poisson JC^* -algebras, *Bull. Braz. Math. Soc.*, 36, 79-97 (2005) · [Zbl 1091.39007](#)
- [21] Park, C., A fixed point approach to the stability of additive functional inequalities in RN-spaces, *Fixed Point Theory*, 12, 429-442 (2011) · [Zbl 1242.39034](#)
- [22] Park, C., Additive ρ -functional inequalities and equations, *J. Math. Inequal.*, 9, 17-26 (2015) · [Zbl 1314.39026](#)
- [23] Park, C., Additive ρ -functional inequalities in non-Archimedean normed spaces, *J. Math. Inequal.*, 9, 397-407 (2015) · [Zbl 1323.39023](#)
- [24] Radu, V., The fixed point alternative and the stability of functional equations, *Fixed Point Theory*, 4, 91-96 (2003) · [Zbl 1051.39031](#)
- [25] Rassias, T. M., On the stability of the linear mapping in Banach spaces, *Proc. Am. Math. Soc.*, 72, 297-300 (1978) · [Zbl 0398.47040](#)
- [26] Ulam, S. M., *A Collection of the Mathematical Problems* (1960), New York: Interscience Publ., New York · [Zbl 0086.24101](#)
- [27] Wang, Z., Stability of two types of cubic fuzzy set-valued functional equations, *Results Math.*, 70, 1-14 (2016) · [Zbl 1352.39025](#)
- [28] Zettl, H., A characterization of ternary rings of operators, *Adv. Math.*, 48, 117-143 (1983) · [Zbl 0517.46049](#)

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