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A computational multi-scale model for the stiffness degradation of short-fiber reinforced plastics subjected to fatigue loading. (English) Zbl 07337801

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Summary: In this work, we investigate a model for the anisotropic and loading-direction dependent stiffness degradation of short-fiber reinforced thermoplastics subjected to high-cycle fatigue loading. Based upon the variational setting of generalized standard materials, we model the stiffness degradation of the matrix in cycle space by a simple isotropic fatigue-damage model with minimum number of model parameters, and consider the fibers to be linear elastic. The stiffness degradation upon cyclic loading is determined, for fixed damage state, by standard linear elastic homogenization. We thoroughly investigate the influence of the involved numerical and material parameters on the effective stiffness degradation of short-fiber reinforced volume elements, and also pay close attention on the dependence on geometric parameters of the composite like fiber-volume fraction and the fiber-orientation state. To enable component-scale simulations, we use a model-order reduction strategy for identifying a suitable macroscopic model. This reduction is possible because our free energy involves polynomial powers in the fields not exceeding two, and our dissipation potential is assumed quadratic. For resolving the balance of linear momentum on the microscopic scale, we rely upon a fast Fourier transform based non-local micromechanics solver. We investigate the clever identification of the modes and demonstrate the capabilities of our model on the microscopic and on the component scale by pertinent numerical examples.

MSC:

74 Mechanics of deformable solids
76 Fluid mechanics

Keywords:

high-cycle fatigue damage; stiffness degradation; short-fiber reinforced materials; FFT-based micromechanics; model-order reduction; computational homogenization

Software:

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