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A complete classification of 3-dimensional quadratic as-regular algebras of type EC. (English)

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Summary: Classification of AS-regular algebras is one of the main interests in noncommutative algebraic geometry. We say that a 3-dimensional quadratic AS-regular algebra is of Type EC if its point scheme is an elliptic curve in \mathbb{P}^2 . In this paper, we give a complete list of geometric pairs and a complete list of twisted superpotentials corresponding to such algebras. As an application, we show that there are only two exceptions up to isomorphism among all 3-dimensional quadratic AS-regular algebras that cannot be written as a twist of a Calabi-Yau AS-regular algebra by a graded algebra automorphism.

MSC:

- 16E65 Homological conditions on associative rings (generalizations of regular, Gorenstein, Cohen-Macaulay rings, etc.)
- 16W50 Graded rings and modules (associative rings and algebras)
- 16S37 Quadratic and Koszul algebras
- 14A22 Noncommutative algebraic geometry
- 16S38 Rings arising from noncommutative algebraic geometry
- 14H52 Elliptic curves

Keywords:

as-regular algebras; geometric algebras; Calabi-Yau algebras; superpotentials; elliptic curves

Full Text: DOI

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