

**Benkhira, El-Hassan; Fakhar, Rachid; Mandyly, Youssef**

**A convergence result and numerical study for a nonlinear piezoelectric material in a frictional contact process with a conductive foundation.** (English) Zbl 07332691

Appl. Math., Praha 66, No. 1, 87-113 (2021)

**Summary:** We consider two static problems which describe the contact between a piezoelectric body and an obstacle, the so-called foundation. The constitutive relation of the material is assumed to be electro-elastic and involves the nonlinear elastic constitutive Hencky's law. In the first problem, the contact is assumed to be frictionless, and the foundation is nonconductive, while in the second it is supposed to be frictional, and the foundation is electrically conductive. The contact is modeled with the normal compliance condition with finite penetration, the regularized Coulomb law, and the regularized electrical conductivity condition. The existence and uniqueness results are provided using the theory of variational inequalities and Schauder's fixed-point theorem. We also prove that the solution of the latter problem converges towards that of the former as the friction and electrical conductivity coefficients converge towards zero. The numerical solutions of the problems are achieved by using a successive iteration technique; their convergence is also established. The numerical treatment of the contact condition is realized using an Augmented Lagrangian type formulation that leads us to use Uzawa type algorithms. Numerical experiments are performed to show that the numerical results are consistent with the theoretical analysis.

**MSC:**

- 35J87 Unilateral problems for nonlinear elliptic equations and variational inequalities with nonlinear elliptic operators
- 74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
- 49J40 Variational inequalities
- 47J25 Iterative procedures involving nonlinear operators
- 74S05 Finite element methods applied to problems in solid mechanics
- 65N55 Multigrid methods; domain decomposition for boundary value problems involving PDEs
- 37M05 Simulation of dynamical systems

**Keywords:**

piezoelectric body; nonlinear elastic constitutive Hencky's law; normal compliance contact condition; Coulomb's friction law; iteration method; augmented Lagrangian; Uzawa block relaxation

**Full Text:** [DOI](#)

**References:**

- [1] Barboteu, M.; Sofonea, M., Analysis and numerical approach of a piezoelectric contact problem, *Ann. Acad. Rom. Sci., Math. Appl.* 1 (2009), 7-30 · [Zbl 1426.74233](#)
- [2] Barboteu, M.; Sofonea, M., Solvability of a dynamic contact problem between a piezoelectric body and a conductive foundation, *Appl. Math. Comput.* 215 (2009), 2978-2991 · [Zbl 1381.74161](#) · [doi:10.1016/j.amc.2009.09.045](#)
- [3] Benkhira, E.-H.; Fakhar, R.; Mandyly, Y., Analysis and numerical approximation of a contact problem involving nonlinear Hencky-type materials with nonlocal Coulomb's friction law, *Numer. Funct. Anal. Optim.* 40 (2019), 1291-1314 · [Zbl 1419.35059](#) · [doi:10.1080/01630563.2019.1600546](#)
- [4] Brézis, H., Équations et inéquations non linéaires dans les espaces vectoriels en dualité, *Ann. Inst. Fourier* 18 (1968), 115-175 French · [Zbl 0169.18602](#) · [doi:10.5802/aif.280](#)
- [5] Essoufi, E.-H.; Benkhira, E.-H.; Fakhar, R., Analysis and numerical approximation of an electro-elastic frictional contact problem, *Adv. Appl. Math. Mech.* 2 (2010), 355-378 · [Zbl 1262.74017](#) · [doi:10.4208/aamm.09-m0980](#)
- [6] Essoufi, E.-H.; Fakhar, R.; Koko, J., A decomposition method for a unilateral contact problem with Tresca friction arising in electro-elastostatics, *Numer. Funct. Anal. Optim.* 36 (2015), 1533-1558 · [Zbl 1333.74081](#) · [doi:10.1080/01630563.2015.1078812](#)
- [7] Han, W., *A Posteriori Error Analysis Via Duality Theory. With Applications in Modeling and Numerical Approximations*, *Advances in Mechanics and Mathematics* 8, Springer, New York (2005) · [Zbl 1081.65065](#) · [doi:10.1007/b101775](#)
- [8] Haslinger, J.; Mäkinen, R., Shape optimization of elasto-plastic bodies under plane strains: Sensitivity analysis and numerical implementation, *Struct. Optim.* 4 (1992), 133-141 · [doi:10.1007/bf01742734](#)

- [9] Lerguet, Z.; Shillor, M.; Sofonea, M., A frictional contact problem for an electro-viscoelastic body, *Electron. J. Differ. Equ.* 2007 (2007), Paper No. 170, 16 pages · [Zbl 1139.74041](#)
- [10] Ouafik, Y., Contribution à l'étude mathématique et numérique des structures piézoélectriques encontact, Ph.D. Dissertation, Perpignan University, Perpignan (2007), Available at <https://tel.archives-ouvertes.fr/tel-00192884> French

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.