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Jordan-Cattaneo waves: analogues of compressible flow. (English) Zbl 07328376

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Summary: We review work of Jordan on a hyperbolic variant of the Fisher-KPP equation, where a shock solution is found and the amplitude is calculated exactly. The Jordan procedure is extended to a hyperbolic variant of the Chafee-Infante equation. Extension of Jordan's ideas to a model for traffic flow are also mentioned. We also examine a diffusive susceptible-infected (SI) model, and generalizations of diffusive Lotka-Volterra equations, including a Lotka-Volterra-Bass competition model with diffusion. For all cases we show how a Jordan-Cattaneo wave may be analysed and we indicate how to find the wavespeeds and the amplitudes. Finally we present details of a fully nonlinear analysis of acceleration waves in a Cattaneo-Christov poroacoustic model.

MSC:

35-XX Partial differential equations
76-XX Fluid mechanics

Keywords:

acceleration waves; shock waves; Jordan-Cattaneo waves; Chaffee-Infante equation; Lotka-Volterra-bass competition model; diffusive SI infection

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