Hounie, J.; **Zugliani, G.**

Tube structures of co-rank 1 with forms defined on compact surfaces. (English) Zbl 1460.58014

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Summary: We study the global solvability of a locally integrable structure of tube type and co-rank 1 by considering a linear partial differential operator $\mathbb{L}$ associated to a general complex smooth closed 1-form c defined on a smooth closed n-manifold. The main result characterizes the global solvability of $\mathbb{L}$ when $n = 2$ in terms of geometric properties of a primitive of a convenient exact pullback of the form $\mathfrak{Im}(c)$ as well as in terms of homological properties of $\mathfrak{Re}(c)$ related to small divisors phenomena. Although the full characterization is restricted to orientable surfaces, some partial results hold true for compact manifolds of any dimension, in particular, the necessity of the conditions, and the equivalence when $\mathfrak{Im}(c)$ is exact. We also obtain informations on the global hypoellipticity of $\mathbb{L}$ and the global solvability of $\mathbb{L}^{n-1}$ – the last non-trivial operator of the complex when M is orientable.

MSC:

58J10 Differential complexes
35A01 Existence problems for PDEs: global existence, local existence, non-existence
35N10 Overdetermined systems of PDEs with variable coefficients

Keywords:

global solvability; complex vector fields; involutive systems; Liouville numbers

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