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**Mixed variational one-dimensional dynamic thermo-viscoplasticity for wave propagation.**

(English) [Zbl 1477.74010](#)

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**Summary:** In view of recent developments in advanced manufacturing and nanomaterial technologies, which have enabled realization of novel material architectures at small-scale, a mixed variational temperature-dependent rate-dependent thermo-viscoplastic continuum element is proposed and developed for elastic, plastic, and thermal wave propagation problems. The variational scheme is based on a Hamiltonian approach, while for the temporal and spatial discretization scheme a discrete calculus of variations approach is adopted. Within this framework, an exponential temperature dependence for material properties and second sound effects is included. The developed framework sets the foundations that can lead to new class of temperature dependent models for capturing thermoplastic behavior and wave propagation of different materials. The robustness of the proposed variational approach, and aspects of coupled dynamic temperature-dependent thermoplastic response are demonstrated through several case studies. Applications include elastic, plastic, and thermal wave propagation and reflection in semi-infinite and finite length space at small temporal scales.

**MSC:**

[74C10](#) Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)

[74F05](#) Thermal effects in solid mechanics

[74J10](#) Bulk waves in solid mechanics

**Keywords:**

rate-dependent elastic-viscoplastic material; space wave propagation; variational Hamiltonian approach; second sound

**Software:**

[Matlab](#)

**Full Text:** [DOI](#)

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