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Mixed displacement and couple stress finite element method for anisotropic centrosymmetric materials. (English) Zbl 07305824

Eur. J. Mech., A, Solids 85, Article ID 104074, 17 p. (2021)

Summary: The classical theory of elasticity is an idealized model of a continuum, which works well for many engineering applications. However, with careful experiments one finds that it may fail in describing behavior in fatigue, at small scales and in structures having high stress concentration factors. Many size-dependent theories have been developed to capture these effects, one of which is the consistent couple stress theory. In this theory, couple stress μ_{ij} is present in addition to force stress σ_{ij} and its tensor form is shown to have skew symmetry. The mean curvature κ_{ij}, which is defined as the skew-symmetric part of the gradient of rotations, is the correct energy conjugate of the couple stress. This mean curvature κ_{ij} and strain e_{ij} together contribute to the elastic energy. The scope of this paper is to extend the work to study anisotropic materials and present a corresponding finite element method. A fully displacement based finite element method for couple stress elasticity requires C^1 continuity. To avoid this, a mixed formulation is presented with primary variables of displacements u_i and couple stress μ_i vectors, both of which require only C^0 continuity. Centrosymmetric classes of materials are considered here for which force stress and strain are decoupled from couple stress and mean curvature in the constitutive relations. Details regarding the numerical implementation are discussed and the effect of couple stress elasticity on anisotropic materials is examined through several computational examples.

MSC:

74-XX Mechanics of deformable solids

Keywords:

consistent couple stress theory; mixed variational formulation; finite element method; anisotropic materials; centrosymmetric materials

Full Text: DOI

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