Danilov, S.; **Kutsenko, A.**

On the geometric origin of spurious waves in finite-volume discretizations of shallow water equations on triangular meshes. (English) Zbl 1453.65245

J. Comput. Phys. 398, Article ID 108891, 19 p. (2019).

Summary: Computational wave branches are common to linearized shallow water equations discretized on triangular meshes. It is demonstrated that for standard finite-volume discretizations these branches can be traced back to the structure of the unit cell of triangular lattice, which includes two triangles with a common edge. Only subsets of similarly oriented triangles or edges possess the translational symmetry of unit cell. As a consequence, discrete degrees of freedom placed on triangles or edges are geometrically different, creating an internal structure inside unit cells. It implies a possibility of oscillations inside unit cells seen as computational branches in the framework of linearized shallow water equations, or as grid-scale noise generally. Adding dissipative operators based on smallest stencils to discretized equations is needed to control these oscillations in solutions. A review of several finite-volume discretization is presented with focus on computational branches and dissipative operators.

MSC:

65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
86A05 Hydrology, hydrography, oceanography
86A10 Meteorology and atmospheric physics
65Z05 Applications to the sciences

Keywords:

triangular meshes; finite volume discretization; computational dispersion branches

Software:

MPAS-Ocean

Full Text: DOI

References:

[1] Arakawa, A.; Lamb, V. R., Computational design of the basic dynamical processes of the UCLA general circulation model, Methods Comput. Phys., 17, 173-265 (1977)

[2] Bernard, P.-E.; Remacle, J.-F.; Legat, V., Modal analysis on unstructured meshes of dispersion properties of the \(P_1^{N C} - P_1\) pair, Ocean Model., 28, 2-11 (2009)

[3] Chen, C.; Liu, H.; Beardsley, R. C., An unstructured grid, finite-volume, three-dimensional, primitive equations ocean model: applications to coastal ocean and estuaries, J. Atmos. Ocean. Technol., 20, 159-186 (2003)

[4] Chen, Q.; Ringler, T.; Gunzburger, M., A co-volume scheme for rotating shallow water equations on conforming non-orthogonal grids, J. Comput. Phys., 240, 174-197 (2013) · Zbl 1426.76346

[5] Comblen, R.; Lambrecht, J.; Remacle, J-F.; Legat, V., Practical evaluation of five partly discontinuous finite element pairs for the non-conservative shallow water equations, Int. J. Numer. Methods Fluids, 63, 701-724 (2010) · Zbl 1423.76220

[6] Cotter, C. J.; Ham, D. A., Numerical wave propagation for the triangular \(P 1_{D G}\)-P2 finite element pair, J. Comput. Phys., 230, 2806-2820 (2011) · Zbl 1316.76019

[7] Cotter, C. J.; Shipton, J., Mixed finite elements for numerical weather prediction, J. Comput. Phys., 231, 7076-7091 (2012) · Zbl 1284.86005

[8] Cotter, C. J.; Thuburn, J., A finite element exterior calculus framework for the rotating shallow-water equations, J. Comput. Phys., 257, 1506-1526 (2014) · Zbl 1351.76054

[9] Cui, H.; Pietrzak, J. D.; Stelling, G. S., A finite volume analogue of the \(P_{N C 1} - P_1\) finite element: with accurate flooding and drying, Ocean Model., 35, 16-30 (2010)

[10] Danilov, S.; Wang, Q.; Losch, M.; Sidorenko, D.; Schröter, J., Modeling ocean circulation on unstructured meshes: comparison of two horizontal discretizations, Ocean Dyn., 58, 365-374 (2008)

[11] Danilov, S., On utility of triangular C-grid type discretization for numerical modeling of large-scale ocean flows, Ocean Dyn.,

60, 6, 1361-1369 (2010)

[12] Danilov, S., Two finite-volume unstructured mesh models for large-scale ocean modeling, Ocean Model., 47, 14-25 (2012)

[13] Danilov, S., Ocean modeling on unstructured meshes, Ocean Model., 69, 195-210 (2013)

[14] Danilov, S.; Sidorenko, D.; Wang, Q.; Jung, T., The Finite-volumE Sea ice - Ocean Model (FESOM2), Geosci. Model Dev., 10, 765-789 (2017)

[15] Fox-Kemper, B.; Menemenlis, D., Can large eddy simulation techniques improve mesoscale rich ocean models?, (Hecht, M. W.; Hasumi, H., Ocean Modeling in an Eddying Regime (2008), American Geophysical Union), 319-337

[16] Fringer, O. B.; Gerritsen, M.; Street, R. L., An unstructured-grid, finite-volume, nonhydrostatic, parallel coastal ocean simulator, Ocean Model., 14, 139-173 (2006)

[17] Gasmann, A., Inspection of hexagonal and triangular C-grid discretizations of the shallow water equations, J. Comput. Phys., 230, 2706-2721 (2011) · Zbl 1316.76069

[18] Gassmann, A., Discretization of generalized Coriolis and friction terms on the deformed hexagonal C-grid, Q. J. R. Meteorol. Soc., 144, 2038-2053 (2018)

[19] Hanert, E.; Le Roux, D. Y.; Legat, V.; Delesnijder, E., An efficient Eulerian finite element method for the shallow water equations, Ocean Model., 10, 115-136 (2005)

[20] Klemp, J. B., Damping characteristics of horizontal Laplacian diffusion filters, Mon. Weather Rev., 145, 4365-4379 (2017)

[21] Korn, P.; Danilov, S., Elementary dispersion analysis of some mimetic discretizations on triangular C-grids, J. Comput. Phys., 330, 156-172 (2017) · Zbl 1380.65220

[22] Korn, P., Formulation of an unstructured grid model for global ocean dynamics, J. Comput. Phys., 339, 525-552 (2017) · Zbl 1380.65275

[23] Korn, P.; Linardakis, L., A conservative discretization of the shallow-water equations on triangular grids, J. Comput. Phys., 375, 871-900 (2018) · Zbl 1416.86003

[24] Kosevich, A. M., The Crystal Lattice Phonons, Solitons, Dislocations, Superlattices (2005), WILEY-VCH Verlag

[25] Melvin, T.; Staniforth, A.; Thuburn, J., Dispersion analysis of the spectral element method, Q. J. R. Meteorol. Soc., 138, 1934-1947 (2012)

[26] Melvin, T., Dispersion analysis of the \(P_n - P_{n - 1}\) (DG) mixed finite-element pair for atmospheric modelling, J. Comput. Phys., 355, 342-365 (2018) · Zbl 1380.65283

[27] Le Roux, D. Y., Spurious inertial oscillations in shallow-water models, J. Comput. Phys., 231, 7959-7987 (2012) · Zbl 1284.86004

[28] Perot, B., Conservation properties of unstructured staggered mesh schemes, J. Comput. Phys., 159, 58-89 (2000) · Zbl 0972.76068

[29] Ringler, T. D.; Thuburn, J.; Klemp, J. B.; Skamarock, W. C., A unified approach to energy conservation and potential vorticity dynamics for arbitrarily-structured C-grids, J. Comput. Phys., 229, 3065-3090 (2010) · Zbl 1307.76054

[30] Ringler, T.; Petersen, M.; Higdon, R.; Jacobsen, D.; Maltrud, M.; Jones, P. W., A multi-resolution approach to global ocean modelling, Ocean Model., 69, 211-232 (2013)

[31] Le Roux, D. Y.; Rostand, V.; Pouliot, B., Analysis of numerically induced oscillations in 2d finite-element shallow-water models Part I: inertia-gravity waves, SIAM J. Sci. Comput., 29, 331-360 (2007) · Zbl 1387.76055

[32] Rostand, V.; Le Roux, D. Y.; Carey, G., Kernel analysis of the discretized finite difference and finite element shallow-water models, SIAM J. Sci. Comput., 31, 531-556 (2008) · Zbl 1191.35025

[33] Szmelter, J.; Smolarkiewicz, P., An edge-based unstructured mesh discretization in geospherical framework, J. Comput. Phys., 229, 4980-4995 (2010) · Zbl 1346.76097

[34] Thuburn, J., Numerical wave propagation on the hexagonal C-grid, J. Comput. Phys., 227, 5836-5858 (2008) · Zbl 1220.76018

[35] Thuburn, J.; Ringer, T. D.; Skamarock, W. C.; Klemp, J. B., Numerical representation of geostrophic modes on arbitrarily structured C-grids, J. Comput. Phys., 228, 8321-8335 (2009) · Zbl 1173.86304

[36] Thuburn, J., Computational modes in weather and climate models (2013)

[37] Ullrich, P. A., Understanding the treatment of waves in atmospheric models. Part I: The shortest resolved waves of the 1D linearized shallow-water equations, Q. J. R. Meteorol. Soc., 140, 1426-1440 (2014)

[38] Walters, R. A.; Hanert, E.; Pietrzak, J.; Le Roux, D. Y., Comparison of unstructured, staggered grid methods for the shallow water equations, Ocean Model., 28, 106-117 (2009)