Mazurenko, Stanislav; **Jauhiainen, Jyrki**; **Valkonen, Tuomo**

Primal-dual block-proximal splitting for a class of non-convex problems. (English)

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ETNA, Electron. Trans. Numer. Anal. 52, 509-552 (2020).

Summary: We develop block structure-adapted primal-dual algorithms for non-convex non-smooth optimisation problems, whose objectives can be written as compositions $G(x) + F(K(x))$ of non-smooth block-separable convex functions G and F with a nonlinear Lipschitz-differentiable operator K. Our methods are refinements of the nonlinear primal-dual proximal splitting method for such problems without the block structure, which itself is based on the primal-dual proximal splitting method of Chambolle and Pock for convex problems. We propose individual step length parameters and acceleration rules for each of the primal and dual blocks of the problem. This allows them to convergence faster by adapting to the structure of the problem. For the squared distance of the iterates to a critical point, we show local $O(1/N), O(1/N^2)$, and linear rates under varying conditions and choices of the step length parameters. Finally, we demonstrate the performance of the methods for the practical inverse problems of diffusion tensor imaging and electrical impedance tomography.

MSC:

90C30 Nonlinear programming
65K10 Numerical optimization and variational techniques
90C48 Programming in abstract spaces

Keywords:

primal-dual algorithms; convex optimization; non-smooth optimization; step length

Software:

ARock; Julia

Full Text: DOI Link

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