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A RBF-based differential quadrature method for solving two-dimensional variable-order time fractional advection-diffusion equation. (English) Zbl 1451.65131

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Summary: Numerical simulation technique of two-dimensional variable-order time fractional advection-diffusion equation is developed in this paper using radial basis function-based differential quadrature method (RBF-DQ). To the best of the authors' knowledge, this is the first application of this method to variable-order time fractional advection-diffusion equations. For the general case of irregular geometries, the meshless local form of RBF-DQ is used and the multiquadric type of radial basis functions is selected for the computations. This approach allows one to define a reconstruction of the local radial basis functions to treat accurately both the Dirichlet and Neumann boundary conditions on the irregular boundaries. The method is validated by the well documented test examples involving variable-order fractional modeling of air pollution. The numerical results demonstrate that the proposed method provides accurate solutions for two-dimensional variable-order time fractional advection-diffusion equations.

MSC:

65M15	Error bounds for initial value and initial-boundary value problems involving PDEs	Cited in **1** Document
35R11	Fractional partial differential equations	
65D12	Numerical radial basis function approximation	

Keywords:

variable-order time fractional; Neumann boundary condition; RBF-differential quadrature method; differential quadrature method; radial basis function

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Matlab; SU2

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