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Decomposition of degenerate Gromov-Witten invariants. (English) Zbl 07283066
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Summary: We prove a decomposition formula of logarithmic Gromov-Witten invariants in a degeneration setting. A one-parameter log smooth family $X \rightarrow B$ with singular fibre over $b_0 \in B$ yields a family $\mathcal{M}(X/B, \beta) \rightarrow B$ of moduli stacks of stable logarithmic maps. We give a virtual decomposition of the fibre of this family over b_0 in terms of rigid tropical maps to the tropicalization of X/B . This generalizes one aspect of known results in the case that the fibre X_{b_0} is a normal crossings union of two divisors. We exhibit our formulas in explicit examples.

MSC:

- 14N35** Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebraic-geometric aspects)
- 14D23** Stacks and moduli problems

Keywords:

logarithmic Gromov-Witten invariant; moduli stack; logarithmic stable map; degeneration; decomposition; tropical curve; tropical map; rigid tropical curve; Artin fan

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