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Good Wannier bases in Hilbert modules associated to topological insulators. (English)

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Summary: For a large class of physically relevant operators on a manifold with discrete group action, we prove general results on the (non-)existence of a basis of well-localized Wannier functions for their spectral subspaces. This turns out to be equivalent to the freeness of a certain Hilbert module over the group C^*-algebra canonically associated with the spectral subspace. This brings into play K-theoretic methods and justifies their importance as invariants of topological insulators in physics.

MSC:

82D20 Statistical mechanical studies of solids
82D40 Statistical mechanical studies of magnetic materials
35Q55 NLS equations (nonlinear Schrödinger equations)
82C20 Dynamic lattice systems (kinetic Ising, etc.) and systems on graphs in time-dependent statistical mechanics

Keywords:

Wannier bases; topological insulators

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References:

[1] Kohn, W., Analytic properties of Bloch waves and Wannier functions, Phys. Rev., 115, 809-821 (1959) · Zbl 0086.45101

[2] des Cloizeaux, J., Energy bands and projection operators in a crystal: Analytic and asymptotic properties, Phys. Rev., 135, 3, 685-697 (1964)

[3] Nenciu, G., Dynamics of band electrons in electric and magnetic fields: Rigorous justification of the effective Hamiltonians, Rev. Mod. Phys., 63, 1, 91-127 (1991)

[4] Brouder, C.; Panati, G.; Calandra, M.; Mourougane, C.; Marzari, N., Exponential localization of Wannier functions in insulators, Phys. Rev. Lett., 98, 046402 (2007)

[5] Kuchment, P., Tight frames of exponentially decaying Wannier functions, J. Phys. A: Math. Theor., 42, 025203 (2009) · Zbl 1154.82028

[6] Monaco, D.; Panati, G.; Pisante, A.; Teufel, S., Optimal decay of Wannier functions in Chern and quantum Hall insulators, Commun. Math. Phys., 359, 61-100 (2018) · Zbl 1400.82249

[7] Cornean, H. D.; Herbst, I.; Nenciu, G., On the construction of composite Wannier functions, Ann. Henri Poincaré, 17, 3361-3398 (2016) · Zbl 1357.82069

[8] Cornean, H. D.; Monaco, D.; Moscolari, M., Parseval frames of exponentially localized magnetic Wannier functions, Commun. Math. Phys., 371, 1179-1230 (2019) · Zbl 07135157

[9] Bradlyn, B.; Elcoro, L.; Cano, J.; Vergniory, M. G.; Wang, Z.; Felser, C.; Aroyo, M. I.; Bernevig, B. A., Topological quantum chemistry, Nature, 547, 7663, 298 (2017)

[10] Bourne, C.; Carey, A. L.; Rennie, A., The bulk-edge correspondence for the quantum Hall effect in Kasparov theory, Lett. Math. Phys., 105, 9, 1253-1273 (2015) · Zbl 1325.81199

[11] Ewert, E. E.; Meyer, R., Coarse geometry and topological phases, Commun. Math. Phys., 366, 3, 1069-1098 (2019) · Zbl 07041901

[12] Kellendonk, J., On the C^*-algebraic approach to topological phases for insulators, Ann. Henri Poincaré, 18, 7, 2251-2300 (2017) · Zbl 1382.82045

[13] Kubota, Y., Controlled topological phases and bulk-edge correspondence, Commun. Math. Phys., 349, 2, 493-525 (2017) · Zbl 1357.82013

[14] Ludewig, M. and Thiang, G. C., "Cobordism invariance of topological edge-following states," arXiv:2001.08339.

[15] Hannabuss, K. C.; Mathai, V.; Thiang, G. C., T-duality simplifies bulk-boundary correspondence: The parametrised case,

Adv. Theor. Math. Phys., 20, 5, 1193-1226 (2016) · Zbl 1359.81181

[16] Prodan, E.; Schulz-Baldes, H., Bulk and Boundary Invariants for Complex Topological Insulators (2016), Springer · Zbl 1342.82002

[17] Roe, J., Comparing analytic assembly maps, Q. J. Math., 53, 241-248 (2002) · Zbl 1014.46045

[18] Gromov, M., Groups of polynomial growth and expanding maps, Publ. Math. Inst. Hautes Études Sci., 53, 53-78 (1981) · Zbl 0474.20018

[19] Freed, D. S.; Moore, G. W., Twisted equivariant matter, Ann. Henri Poincaré, 14, 8, 1927-2023 (2013) · Zbl 1286.81109

[20] Thiang, G. C., On the K-theoretic classification of topological phases of matter, Ann. Henri Poincaré, 17, 4, 757-794 (2016) · Zbl 1344.81144

[21] Bellissard, J., K-theory of C^*-algebras in solid state physics, Statistical Mechanics and Field Theory: Mathematical Aspects, 99-156 (1986), Springer

[22] For two C^*-algebras \(A_1 \subset \mathcal{L}(\mathcal{H}_1)\) and \(A_2 \subset \mathcal{L}(\mathcal{H}_2)\), their algebraic tensor product sits inside \(\mathcal{L}(\mathcal{H}_1 \otimes \mathcal{H}_2)\) and its completion therein is the spatial tensor product (see Appendix T.5 of Ref. 27 for details of this and other possible C^*-algebra tensor products). In our case where Γ has polynomial growth and is hence amenable, its group C^*-algebra is nuclear. Hence, for any C^*-algebra A, there is actually a unique C^*-algebra tensor product \(A \otimes C_r^*(\operatorname{\Gamma})\). For details, see Ref. 49 (Example 2.6.6, and Theorems 2.6.8 and 3.8.7).

[23] Jolissaint, P., Rapidly decreasing functions in reduced C^*-algebras of groups, Trans. Am. Math. Soc., 317, 1, 167-196 (1990) · Zbl 0711.46054

[24] Ji, R., Smooth dense subalgebras of reduced group C^*-algebras, Schwartz cohomology of groups, and cyclic cohomology, J. Funct. Anal., 107, 1, 1-33 (1992) · Zbl 0787.46043

[25] Valette, A. (2002), Birkhäuser

[26] Lance, E. C., Hilbert C^*-Modules: A Toolkit for Operator Algebraists (1995), Cambridge University Press

[27] Wegge-Olsen, N. E., K-Theory and C^*-Algebras (1993), Oxford University Press

[28] Gruber, M. J., Noncommutative Bloch theory, J. Math. Phys., 42, 6, 2438-2465 (2001) · Zbl 1016.81061

[29] Brüning, J.; Sunada, T., On the spectrum of periodic elliptic operators, Nagoya Math. J., 126, 159-171 (1992) · Zbl 0759.35016

[30] Mathai, V.; Marcolli, M., Twisted index theory on good orbifolds, I: Noncommutative Bloch theory, Commun. Contemp. Math., 1, 4, 553-587 (1999) · Zbl 0959.58035

[31] De Nittis, G.; Panati, G., The topological Bloch-Floquet transform and some applications, Operator Theory: Advances and Applications, 67-105 (2012), Springer · Zbl 1270.81087

[32] Druţu, C.; Kapovich, M., Geometric Group Theory (2018), American Mathematical Society

[33] Han, D.; Kornelson, K.; Larson, D.; Weber, E., Frames for Undergraduates (2007), American Mathematical Society · Zbl 1143.42001

[34] Cheeger, J.; Gromov, M.; Taylor, M., Finite propagation speed, kernel estimates for functions of the Laplace operator, and the geometry of complete Riemannian manifolds, J. Differ. Geom., 17, 1, 15-53 (1982) · Zbl 0493.53035

[35] Engel, A., Rough index theory on spaces of polynomial growth and contractibility, J. Noncommutative Geom., 13, 2, 617-666 (2019) · Zbl 1436.58019

[36] Taylor, M., Partial Differential Equations I: Basic Theory (2011), Springer, Applied Mathematical Sciences · Zbl 1206.35002

[37] Bär, C.; Ginoux, N.; Pfäeffle, F., Wave Equations on Lorentzian Manifolds and Quantization (2007), EMS Publishing House

[38] Panati, G., Triviality of Bloch and Bloch-Dirac bundles, Ann. Henri Poincaré, 8, 995-1011 (2007) · Zbl 1375.81102

[39] Kitaev, A., Periodic table for topological insulators and superconductors, AIP Conf. Proc., 1134, 1, 22-30 (2009) · Zbl 1180.82221

[40] Read, N., Compactly supported Wannier functions and algebraic K-theory, Phys. Rev. B, 95, 11, 115309 (2017)

[41] Baum, P.; Connes, A.; Higson, N., Classifying space for proper actions and K-theory of group C^*-algebras, C^*-Algebras: 1943-1993 (San Antonio, TX, 1993), 240-291 (1994), American Mathematical Society · Zbl 0830.46061

[42] Anderson, J.; Paschke, W., The rotation algebra, Houston J. Math., 15, 1, 1-26 (1989) · Zbl 0703.22005

[43] Gomi, K.; Thiang, G. C., Crystallographic bulk-edge correspondence: Glide reflections and twisted mod 2 indices, Lett. Math. Phys., 109, 4, 857-904 (2019) · Zbl 1436.19011

[44] Rieffel, M. A., Dimension and stable rank in the K-theory of C^*-algebras, Proc. London Math. Soc., s3-4, 2, 301-333 (1983) · Zbl 0533.46046

[45] Blackadar, B., K-Theory for Operator Algebras (1998), Cambridge University Press

[46] Gomi, K., A variant of K-theory and topological T-duality for real circle bundles, Commun. Math. Phys., 334, 923-975 (2015) · Zbl 1320.19001

[47] Kleinert, H., Gauge Fields in Condensed Matter (1989), World Scientific

[48] Sudo, T., Stable rank of C^*-algebras of continuous fields, Tokyo J. Math., 28, 1, 173-188 (2015) · Zbl 1091.46033

[49] Brown, N. P.; Ozawa, N., C^*-Algebras and Finite-Dimensional Approximations (2008), American Mathematical Society · Zbl 1160.46001