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**A note on representable group topologies.** (English) [Zbl 07273625](#)

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Summary: We study natural topologies in the sense of Debreu in the poset of topological group topologies on a topological group. We call this kind of topologies  $g$ -topologies. To be precise, groups admitting a non-totally disconnected  $g$ -natural topology as well as the non-totally disconnected  $g$ -topologies are identified. Moreover, the non-totally disconnected  $g$ -representable topologies as well as the total orders inducing non-totally disconnected group topologies are characterized. It is worth noting that our framework is more general than the usual one in representation theory: we assume no translation invariant properties. We also deal with some questions concerning order and topological algebra related to the semicontinuous representation property (SRP): we establish some results related to the Sorgenfrey line and SRP (some of them connected to the Proper Forcing Axiom (PFA)) and, we characterize  $\sigma$ -compact and (locally) precompact groups which satisfy SRP.

For the entire collection see [\[Zbl 1446.91009\]](#).

**MSC:**

[22A05](#) Structure of general topological groups

[54-XX](#) General topology

**Keywords:**

order preserving real-valued functions; continuous and additive order preserving real-valued functions; representability of an ordered structure; (locally pseudocompact) topological group; linear order; totally ordered group; algebraically orderable group; topologically orderable group; non-Archimedean metric; the Sorgenfrey line; cardinal function; locally pseudocompact group

**Full Text:** [DOI](#)

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