Al-Salman, Ahmad; AlSharawi, Ziyad; Kallel, Sadok
Extension, embedding and global stability in two dimensional monotone maps. (English)
Zbl 1451.39009
Discrete Contin. Dyn. Syst., Ser. B 25, No. 11, 4257-4276 (2020).

Summary: We consider the general second order difference equation $x_{n+1} = F(x_n, x_{n-1})$ in which F is continuous and of mixed monotonicity in its arguments. In equations with negative terms, a persistent set can be a proper subset of the positive orthant, which motivates studying global stability with respect to compact invariant domains. In this paper, we assume that F has a semi-convex compact invariant domain, then make an extension of F on a rectangular domain that contains the invariant domain. The extension preserves the continuity and monotonicity of F. Then we use the embedding technique to embed the dynamical system generated by the extended map into a higher dimensional dynamical system, which we use to characterize the asymptotic dynamics of the original system. Some illustrative examples are given at the end.

MSC:

39A22 Growth, boundedness, comparison of solutions to difference equations
39A30 Stability theory for difference equations
39A10 Additive difference equations
37E30 Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces

Keywords:
invariant domain; embedding; monotone maps; global stability

Full Text: DOI

References:

[1] R. Abu-Saris; Z. AlSharawi; M. B. H. Rhouma, The dynamics of some discrete models with delay under the effect of constant yield harvesting, Chaos Solitons Fractals, 54, 26-38 (2013) · Zbl 1341.92056

[2] Z. AlSharawi, A global attractor in some discrete contest competition models with delay under the effect of periodic stocking, Abstr. Appl. Anal., (2013), Art. ID 101649, 7 pp. · Zbl 1297.39018

[3] A. M. Amleh; E. Camouzis; G. Ladas, On second-order rational difference equation. ⊠, J. Difference Equ. Appl., 13, 969-1004 (2007) · Zbl 1131.39005

[4] A. M. Amleh; E. Camouzis; G. Ladas, On the dynamics of a rational difference equation. ⊠, Int. J. Difference Equ., 3, 195-225 (2008)

[5] E. Camouzis and G. Ladas, Dynamics of Third-order Rational Difference Equations with Open Problems and Conjectures, Advances in Discrete Mathematics and Applications, vol. 5, Chapman \& Hall/CRC, Boca Raton, FL, 2008. · Zbl 1129.39002

[6] E. Camouzis; G. Ladas, When does local asymptotic stability imply global attractivity in rational equations?, J. Difference Equ. Appl., 12, 863-885 (2006) · Zbl 1105.39001

[7] W. A. Coppel, The solution of equations by iteration, Proc. Cambridge Philos. Soc., 51, 41-43 (1955) · Zbl 0064.12303

[8] J.-L. Gouzé; K. P. Hadeler, Monotone flows and order intervals, Nonlinear World, 1, 23-34 (1994) · Zbl 0803.65076

[9] E. A. Grove and G. Ladas, Periodicities in Nonlinear Difference Equations, Advances in Discrete Mathematics and Applications, vol. 4, Chapman \& Hall/CRC, Boca Raton, FL, 2005. · Zbl 1078.39009

[10] V. L. Kocić and G. Ladas, Global Behavior of Nonlinear Difference Equations of Higher Order with Applications, Mathematics and its Applications, vol. 256, Kluwer Academic Publishers Group, Dordrecht, 1993. · Zbl 0787.39001

[11] M. R. S. Kulenović and G. Ladas, Dynamics of Second Order Rational Difference Equations, With Open Problems and Conjectures, Chapman \& Hall/CRC, Boca Raton, FL, 2002. · Zbl 0981.39011

[12] M. R. S. Kulenović, G. Ladas, L. F. Martins and I. W. Rodrigues, The dynamics of \(x_{n+1} = \frac{\alpha+\beta x_n}{A+Bx_n+Cx_{n-1}} \): Facts and conjectures, Comput. Math. Appl., 45 (2003), 1087-1099, 2003. · Zbl 1077.39004

[13] M. R. S. Kulenović; G. Ladas; W. S. Sizer, On the recursive sequence \(x_{n+1} = (\alpha x_n+\beta x_{n-1})/(\gamma x_n+\delta x_{n-1})\), Math. Sci. Res. Hot-Line, 2, 1-16 (1998) · Zbl 0960.39502

[14] M. R. S. Kulenović; O. Merino, A note on unbounded solutions of a class of second order rational difference equations, J.

Difference Equ. Appl., 12, 777-781 (2006) · Zbl 1107.39007

[15] M. R. S. Kulenović; O. Merino, Global bifurcation for discrete competitive systems in the plane, Discrete Contin. Dyn. Syst. Ser. B, 12, 133-149 (2009) · Zbl 1175.37058

[16] M. R. S. Kulenović; O. Merino, Invariant manifolds for competitive discrete systems in the plane, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 20, 2471-2486 (2010) · Zbl 1202.37027

[17] W. A. J. Luxemburg and A. C. Zaanen, Riesz spaces. Vol. I, North-Holland Mathematical Library, North-Holland Publishing Co., Amsterdam-London; American Elsevier Publishing Co., New York, 1971. · Zbl 0231.46014

[18] G. Nyerges, A note on a generalization of Pielou's equation, J. Difference Equ. Appl., 14, 563-565 (2008) · Zbl 1151.39008

[19] H. Sedaghat, Nonlinear Difference Equations. Theory with Applications to Social Science Models, Mathematical Modelling: Theory and Applications, vol. 15, Kluwer Academic Publishers, Dordrecht, 2003. · Zbl 1020.39007

[20] H. L. Smith, The discrete dynamics of monotonically decomposable maps, J. Math. Biol., 53, 747-758 (2006) · Zbl 1118.65057

[21] H. L. Smith, Global stability for mixed monotone systems, J. Difference Equ. Appl., 14, 1159-1164 (2008) · Zbl 1162.39009