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Averaging principles for nonautonomous two-time-scale stochastic reaction-diffusion equations with jump. (English) Zbl 1451.60073

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Summary: In this paper, we aim to develop the averaging principle for a slow-fast system of stochastic reaction-diffusion equations driven by Poisson random measures. The coefficients of the equation are assumed to be functions of time, and some of them are periodic or almost periodic. Therefore, the Poisson term needs to be processed, and a new averaged equation needs to be given. For this reason, the existence of time-dependent evolution family of measures associated with the fast equation is studied and proved that it is almost periodic. Next, according to the characteristics of almost periodic functions, the averaged coefficient is defined by the evolution family of measures, and the averaged equation is given. Finally, the validity of the averaging principle is verified by using the Khasminskii method.

MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis)
35K57 Reaction-diffusion equations
35R60 PDEs with randomness, stochastic partial differential equations

Full Text: DOI

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