Adamaszek, Michał; **Adams, Henry**; **Gasparovic, Ellen**; **Gommel, Maria**; **Purvine, Emilie**; **Sazdanovic, Radmila**; **Wang, Bei**; **Wang, Yusu**; **Ziegelmeier, Lori**

On homotopy types of Vietoris-Rips complexes of metric gluings. (English) Zbl 1455.55005

J. Appl. Comput. Topol. 4, No. 3, 425-454 (2020).

The Vietoris-Rips complex is a fundamental tool in persistent homology theory or Topological Data Analysis (TDA). This complex can recover topological features of a sample underlying the data. Indeed, it was proved that if the underlying space is a closed Riemannian manifold M, the scale parameter is sufficiently small, and a sample is sufficiently close to M, then the Vietoris-Rips complex of the sample is homotopy equivalent to M [*J.-C. Hausmann*, Ann. Math. Stud. 138, 175–188 (1995; Zbl 0928.55003); *J. Latschev*, Arch. Math. 77, No. 6, 522–528 (2001; Zbl 1001.53026)]). In this paper, the authors study the Vietoris-Rips complexes of glued metric spaces at all scale parameters. In particular, it is proved that the Vietoris-Rips complex of the wedge sum of two pointed metric spaces $\mathrm{VR}(X \vee Y; r)$ is homotopy equivalent to the wedge sum of the Vietoris-Rips complexes $\mathrm{VR}(X; r) \vee \mathrm{VR}(Y; r)$ for all $r > 0$. More generally, the Vietoris-Rips complex of the glued space of two metric spaces along a common isometric subset is studied. These results enable us to compute the persistent homology of a glued space completely. Čech analogies are also studied. As an application, the Vietoris-Rips complexes of glued metric graphs are discussed.

Reviewer: Yuichi Ike (Kawasaki)

MSC:

55N31 Persistent homology and applications, topological data analysis
55U10 Simplicial sets and complexes in algebraic topology
68T09 Computational aspects of data analysis and big data
55P15 Classification of homotopy type
05E45 Combinatorial aspects of simplicial complexes

Keywords:

Vietoris-Rips complex; Čech complex; metric space gluings; wedge sums; metric graphs; persistent homology

Full Text: DOI

References:

[1] Aanjaneya, M.; Chazal, F.; Chen, D.; Glisse, M.; Guibas, L.; Morozov, D., Metric graph reconstruction from noisy data, Int. J.Comput. Geom. Appl., 22, 4, 305-325 (2012)

[2] Adamaszek, M., Clique complexes and graph powers, Israel J. Math., 196, 1, 295-319 (2013) · Zbl 1275.05041

[3] Adamaszek, M.; Adams, H., The Vietoris-Rips complexes of a circle, Pac. J. Math., 290, 1-40 (2017) · Zbl 1366.05124

[4] Adamaszek, M.; Adams, H.; Frick, F.; Peterson, C.; Previte-Johnson, C., Nerve complexes of circular arcs, Discrete Comput. Geom., 56, 2, 251-273 (2016) · Zbl 1354.05149

[5] Adamaszek, M., Adams, H., Gasparovic, E., Gommel, M., Purvine, E., Sazdanovic, R., Wang, B., Wang, Y., Ziegelmeier, L.: Vietoris-Rips and Čech complexes of metric gluings. In B. Speckmann and C. D. Tóth, editors, 34th International Symposium on Computational Geometry, volume 99 of Leibniz International Proceedings in Informatics (LIPIcs), pp. 3:1-3:15, Dagstuhl, Germany. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik (2018) · Zbl 1422.55037

[6] Adams, H., Heim M., Peterson, C.: Metric thickenings and group actions (2019). arXiv preprint arXiv:1911.00732

[7] Babson, E.; Kozlov, DN, Complexes of graph homomorphisms, Israel J. Math., 152, 1, 285-312 (2006) · Zbl 1205.52009

[8] Barmak, JA, On Quillen's Theorem A for posets, J. Comb. Theory Ser. A, 118, 8, 2445-2453 (2011) · Zbl 1234.05237

[9] Barmak, JA; Minian, EG, Simple homotopy types and finite spaces, Adv. Math., 218, 87-104 (2008) · Zbl 1146.57034

[10] Biswal, B.; Yetkin, FZ; Haughton, VM; Hyde, JS, Functional connectivity in the motor cortex of resting human brain using echo-planar MRI, Magn. Reson. Med., 34, 537-541 (1995)

[11] Björner, A., Topological methods, Handb. Comb., 2, 1819-1872 (1995) · Zbl 0851.52016

[12] Bridson, MR; Haefliger, A., Metric Spaces of Non-Positive Curvature (1999), Berlin: Springer, Berlin

[13] Burago, D.; Burago, Y.; Ivanov, S., A Course in Metric Geometry (2001), Providence: American Mathematical Society, Providence · Zbl 0981.51016

[14] Carlsson, G., Topology and data, Bull. Am. Math. Soc., 46, 2, 255-308 (2009) · Zbl 1172.62002

[15] Chachólski, W., Jin, A., Scolamiero, M., Tombari, F.: Homotopical decompositions of simplicial and Vietoris Rips complexes (2020). arXiv preprint arXiv:2002.03409

[16] Chan, JM; Carlsson, G.; Rabadan, R., Topology of viral evolution, Proc. Natl. Acad. Sci., 110, 46, 18566-18571 (2013) · Zbl 1292.92014

[17] Chazal, F.; de Silva, V.; Glisse, M.; Oudot, S., The Structure and Stability of Persistence Modules (2016), Berlin: Springer, Berlin · Zbl 1362.55002

[18] Chazal, F.; de Silva, V.; Oudot, S., Persistence stability for geometric complexes, Geom. Dedic., 173, 1-22 (2013)

[19] Chazal, F.; Cohen-Steiner, D.; Guibas, LJ; Mémoli, F.; Oudot, SY, Gromov-Hausdorff stable signatures for shapes using persistence, Comput. Graph. Forum, 28, 5, 1393-1403 (2009)

[20] Dong, F.; Koh, K-M; Teo, KL, Chromatic Polynomials and Chromaticity of Graphs (2005), Singapore: World Scientific, Singapore

[21] Edelsbrunner, H.; Harer, JL, Computational Topology: An Introduction (2010), Providence: American Mathematical Society, Providence

[22] Gasparovic, E., Gommel, M., Purvine, E., Sazdanovic, R., Wang, B., Wang, Y., Ziegelmeier, L.: A complete characterization of the one-dimensional intrinsic Čech persistence diagrams for metric graphs. In: Research in Computational Topology (2018) · Zbl 1422.55037

[23] Hatcher, A., Algebraic Topology (2002), Cambridge: Cambridge University Press, Cambridge

[24] Hausmann, J-C, On the Vietoris-Rips complexes and a cohomology theory for metric spaces, Ann. Math. Stud., 138, 175-188 (1995) · Zbl 0928.55003

[25] Kozlov, DN, Combinatorial Algebraic Topology, volume 21 of Algorithms and Computation in Mathematics (2008), Berlin: Springer, Berlin

[26] Kuchment, P., Quantum graphs I. Some basic structures, Waves Random Media, 14, 1, S107-S128 (2004) · Zbl 1063.81058

[27] Latschev, J., Vietoris-Rips complexes of metric spaces near a closed Riemannian manifold, Arch. Math., 77, 6, 522-528 (2001) · Zbl 1001.53026

[28] Lesnick, M.; Rabadan, R.; Rosenbloom, DI, Quantifying genetic innovation: mathematical foundations for the topological study of reticulate evolution, SIAM J. Appl. Algebra Geom., 4, 1, 141-184 (2020) · Zbl 1433.92030

[29] Matoušek, J., LC reductions yield isomorphic simplicial complexes, Contrib. Discrete Math., 3, 2, 37-39 (2008) · Zbl 1191.52011

[30] Oudot, S., Solomon, E.: Barcode embeddings for metric graphs (2017). arXiv preprint arXiv:1712.03630

[31] Quillen, DD; Bass, H., Higher algebraic k-theory: I, Higher K-Theories, 85-147 (1973), Berlin: Springer, Berlin · Zbl 0292.18004

[32] Sousbie, T.; Pichon, C.; Kawahara, H., The persistent cosmic web and its filamentary structure—II. Illustrations, Mon. Not. R. Astron. Soc., 414, 1, 384-403 (2011)

[33] Turner, K., Rips filtrations for quasimetric spaces and asymmetric functions with stability results, Algebr. Geom. Topol., 19, 3, 1135-1170 (2019) · Zbl 1434.55002

[34] Virk, Ž., 1-dimensional intrinsic persistence of geodesic spaces, J. Topol. Anal., 12, 1-39 (2018)