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Desingularization of vortex rings in 3 dimensional Euler flows. (English) Zbl 1451.35120
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Summary: In this paper, we are concerned with nonlinear desingularization of steady vortex rings of three-dimensional incompressible Euler fluids. We focus on the case when the vorticity function has a simple discontinuity, which corresponding to a jump in vorticity at the boundary of the cross-section of the vortex ring. Using the vorticity method, we construct a family of steady vortex rings which constitute a desingularization of the classical circular vortex filament in several kinds of domains. The precise localization of the asymptotic singular vortex filament is proved to depend on the circulation and the velocity at far fields of the vortex ring, and the geometry of the domains. Some qualitative and asymptotic properties are also established.

MSC:

35Q31 Euler equations
76B47 Vortex flows for incompressible inviscid fluids
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

[incompressible Euler system](#); [vortex rings](#); [inviscid limit](#)

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