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Global Gevrey solvability for a class of perturbations of involutive systems. (English)

Zbl 1454.35050
J. Differ. Equations 270, 994-1018 (2021).

In this article, the author is interested in the global solvability, in the Gevrey meaning, on the n-dimensional torus $\mathbb{T}^n$ of the system

$$(L_j = \partial_{t_j} + a_j(t)\partial_x + b_j(t))_{1\leq j\leq n} \in \mathbb{T}_t^n \times S_x^1,$$

where $a_j \in G^s(\mathbb{T}^n, \mathbb{R})$ and $b_j \in G^s(\mathbb{T}^n)$ are both s-Gevrey on the torus $\mathbb{T}^n$, and where $\sum_{j=1}^{n} a_j dt_j$ and $\sum_{j=1}^{n} b_j dt_j$ are both closed. More precisely, he focuses in the following question: supposing that the system

$$(\partial_{t_j} + a_j(t)\partial_x)_{1\leq j\leq n}$$

is globally s-solvable, when the system $(L_j)_{1\leq j\leq n}$ is also globally s-solvable?

To do that, the author first reduces the study of the system $(L_j)_{1\leq j\leq n}$ to the study of a system whose the principal part has constant coefficients. Then, he generalizes the results of [*G. Petronilho* and *S. L. Zani*, J. Differ. Equations 244, No. 9, 2372–2403 (2008; Zbl 1155.35010)] in order to characterize the global s-solvability of the latter system.

Reviewer: Pascal Remy (Carrières-sur-Seine)

MSC:

35F05 Linear first-order PDEs
35N10 Overdetermined systems of PDEs with variable coefficients

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