Ghoshal, Shyam Sundar; **Jana, Animesh**; **Towers, John D.**

Convergence of a Godunov scheme to an Audusse-Perthame adapted entropy solution for conservation laws with BV spatial flux. (English) Zbl 1451.65107

Numer. Math. 146, No. 3, 629-659 (2020).

Summary: In this article we consider the initial value problem for a scalar conservation law in one space dimension with a spatially discontinuous flux. There may be infinitely many flux discontinuities, and the set of discontinuities may have accumulation points. Thus the existence of traces cannot be assumed. In [*E. Audusse* and *B. Perthame*, Proc. R. Soc. Edinb., Sect. A, Math. 135, No. 2, 253–266 (2005; Zbl 1071.35079)] proved a uniqueness result that does not require the existence of traces, using adapted entropies. We generalize the Godunov-type scheme of *Adimurthi* et al. [SIAM J. Numer. Anal. 42, No. 1, 179–208 (2004; Zbl 1081.65082)] for this problem with the following assumptions on the flux function, (i) the flux is BV in the spatial variable and (ii) the critical point of the flux is BV as a function of the space variable. We prove that the Godunov approximations converge to an adapted entropy solution, thus providing an existence result, and extending the convergence result of Adimurthi, Jaffré and Gowda.

MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
35L65 Hyperbolic conservation laws
35B44 Blow-up in context of PDEs
35A01 Existence problems for PDEs: global existence, local existence, non-existence
65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
76M20 Finite difference methods applied to problems in fluid mechanics

Keywords:

Godunov scheme; convergence

Full Text: DOI

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