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**Percolative properties of Brownian interlacements and its vacant set.** (English) Zbl 07268516  
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Summary: In this article, we investigate the percolative properties of Brownian interlacements, a model introduced by Sznitman (*Bull Braz Math Soc New Ser* 44(4):555-592, 2013), and show that: the interlacement set is “well-connected”, i.e., any two “sausages” in  $d$ -dimensional Brownian interlacements,  $d \geq 3$ , can be connected via no more than  $\lceil (d-4)/2 \rceil$  intermediate sausages almost surely; while the vacant set undergoes a non-trivial percolation phase transition when the level parameter varies.

**MSC:**

[60J65](#) Brownian motion

[60K35](#) Interacting random processes; statistical mechanics type models; percolation theory

[82B43](#) Percolation

**Keywords:**

[Brownian interlacements](#); [random interlacements](#); [Wiener sausage](#); [percolation](#)

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**References:**

- [1] Ahlberg, D.; Tassion, V.; Teixeira, A., Sharpness of the phase transition for continuum percolation in  $(\mathbb{R}^2)$ , *Probab. Theory Relat. Fields*, 172, 525-581 (2018) · [Zbl 1404.60143](#)
- [2] Albeverio, S.; Zhou, XY, Intersections of random walks and Wiener sausages in four dimensions, *Acta Appl. Math.*, 45, 2, 195-237 (1996) · [Zbl 0858.60043](#)
- [3] Černý, J.; Popov, S., On the internal distance in the interlacement set, *Electron. J. Probab.*, 17, 29, 1-25 (2012) · [Zbl 1245.60090](#)
- [4] Černý, J.; Teixeira, A., From random walk trajectories to random interlacements, *Ens. Mat.*, 23, 1-78 (2012) · [Zbl 1269.60002](#)
- [5] Černý, J.; Teixeira, A., Random walks on torus and random interlacements: macroscopic coupling and phase transition, *Ann. Appl. Probab.*, 26, 5, 2883-2914 (2016) · [Zbl 1353.60083](#)
- [6] Cox, DR, *Renewal Theory* (1962), London: Methuen & Co., London
- [7] Daley, DJ; Vere-Jones, D., *An Introduction to the Theory of Point Processes* (2008), Berlin: Springer, Berlin
- [8] Drewitz, A.; Ráth, B.; Sapozhnikov, A., *An Introduction to Random Interlacements* (2014), Berlin: Springer, Berlin · [Zbl 1304.60008](#)
- [9] Elias, O., Tykesson, J.: Visibility in the vacant set of the Brownian interlacements and the Brownian excursion process. [arXiv:1709.09052](#) · [Zbl 07116306](#)
- [10] Goodman, J.; den Hollander, F., Extremal geometry of a Brownian porous medium, *Probab. Theory Relat. Fields*, 160, 1-2, 127-174 (2014) · [Zbl 1327.60037](#)
- [11] Grimmett, G., *Percolation* (1999), Berlin: Springer, Berlin
- [12] Hara, T.; van der Hofstad, R.; Slade, G., Critical two-point functions and the lace expansion for spread-out high-dimensional percolation and related models, *Ann. Probab.*, 31, 1, 349-408 (2003) · [Zbl 1044.82006](#)
- [13] Li, X.; Sznitman, A-S, Large deviations for occupation time profiles of random interlacements, *Probab. Theory Relat. Fields*, 161, 1, 309-350 (2015) · [Zbl 1314.60078](#)
- [14] Matheron, G., *Random Sets and Integral Geometry* (1975), New York: Wiley, New York · [Zbl 0321.60009](#)
- [15] Nash, SW, An extension of the Borel-Cantelli lemma, *Ann. Math. Stat.*, 25, 165-167 (1954) · [Zbl 0055.12302](#)
- [16] Paley, REAC; Zygmund, A., A note on analytic functions in the unit circle, *Proc. Camb. Philos. Soc.*, 28, 266-272 (1932) · [Zbl 58.1076.03](#)
- [17] Erhard, D.; Poisat, J., Asymptotics of the critical time in Wiener sausage percolation with a small radius, *ALEA, Lat. Am. J. Probab. Math. Stat.*, 13, 1, 417-445 (2016) · [Zbl 1346.60138](#)
- [18] Popov, S.; Teixeira, A., Soft local times and decoupling of random interlacements, *J. Eur. Math. Soc.*, 17, 10, 2545-2593 (2015) · [Zbl 1329.60342](#)
- [19] Port, S.; Stone, C., *Brownian Motion and Classical Potential Theory* (1978), New York: Academic Press, New York · [Zbl 0413.60067](#)

- [20] Procaccia, E.; Tykesson, J., Geometry of the random interlacement, *Electron. Commun. Probab.*, 16, 47, 528-544 (2011) · [Zbl 1254.60018](#)
- [21] Ráth, B., A short proof of the phase transition for the vacant set of random interlacements, *Electron. Commun. Probab.*, 20, 3, 1-11 (2015) · [Zbl 1307.60146](#)
- [22] Ráth, B.; Sapozhnikov, A., On the transience of random interlacements, *Electron. Commun. Probab.*, 16, 35, 379-391 (2011) · [Zbl 1231.60115](#)
- [23] Ráth, B.; Sapozhnikov, A., Connectivity properties of random interlacement and intersection of random walks, *ALEA, Lat. Am. Probab. Math. Stat.*, 9, 67-83 (2012) · [Zbl 1277.60182](#)
- [24] Ráth, B.; Sapozhnikov, A., The effect of small quenched noise on connectivity properties of random interlacements, *Electron. J. Probab.*, 18, 4, 1-20 (2013) · [Zbl 1347.60132](#)
- [25] Sznitman, A-S, *Brownian Motion, Obstacles and Random Media* (1998), Berlin: Springer, Berlin
- [26] Sznitman, A-S, Upper bound on the disconnection time of discrete cylinders and random interlacements, *Ann. Probab.*, 37, 5, 1715-1746 (2009) · [Zbl 1179.60025](#)
- [27] Sznitman, A-S, Vacant set of random interlacements and percolation, *Ann. Math.*, 171, 2039-2087 (2010) · [Zbl 1202.60160](#)
- [28] Sznitman, A-S, Decoupling inequalities and interlacement percolation on  $(\mathcal{G} \times \mathbb{Z})$ , *Invent. Math.*, 187, 645-706 (2012) · [Zbl 1277.60183](#)
- [29] Sznitman, A-S, On scaling limits and Brownian interlacements, *Bull. Braz. Math. Soc., New Ser.*, 44, 4, 555-592 (2013) · [Zbl 1303.60022](#)
- [30] Teixeira, A., On the uniqueness of the infinite cluster of the vacant set of random interlacements, *Ann. Appl. Probab.*, 19, 1, 454-466 (2009) · [Zbl 1158.60046](#)
- [31] Teixeira, A.; Windisch, D., On the fragmentation of a torus by random walk, *Commun. Pure Appl. Math.*, 64, 12, 1599-1646 (2011) · [Zbl 1235.60143](#)

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