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Symmetry properties, conservation laws, reduction and numerical approximations of time-fractional cylindrical-Burgers equation. (English) Zbl 07263878

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Summary: In this paper the Lie group analysis of the time-fractional cylindrical Burgers equation (time-FCB), which is a fundamental PDE occurring in various areas of applied mathematics, such as fluid mechanics, non-linear acoustics, gas dynamics, traffic flow and etc. is given. For this purpose the Riemann-Liouville derivative is used to implement the Lie algorithm for finding the symmetry operators. A reduced form of the equation is given by using the similarity variables obtained from a symmetry and Erdelyi-Kober operator. In the next step conservation laws are derived via a generalization of Noether's theorem. Finally the Chebyshev wavelets for time-fractional differential equations (FDEs) is applied for solving the considered equation.

MSC:

00 General and overarching topics; collections

Keywords:

symmetry analysis; fractional derivatives; Chebyshev wavelets; fractional conservation laws; Noether's theorem

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References:

[1] Srivastava, V. K.; Awasthi, M. K.; Tamsir, M., RDTM Solution of Caputo time fractional-order hyperbolic telegraph equation, AIP Adv, 3, 032142 (2013)

[2] Abdel-Gawad Hamdy, I.; Osman, M. S., On the variational approach for analyzing the stability of solutions of evolution equations, Kyungpook Math J, 53, 661-680 (2013)

[3] Osman, M. S., Multiwave solutions of time-fractional (2+1)-dimensional Nizhnik-Novikov-Veselov equations, Pramana J Phys, 88, 67 (2017)

[4] Sahadevan, R.; Bakkyaraj, T., Invariant analysis of time fractional generalized burgers and Korteweg-de Vries equations, J Math Anal Appl, 393, 341-347 (2012)

[5] Gazizov, R. K.; Kasatkin, A. A.; Lukashchuk, S. Y., Continuous transformation groups of fractional differential equations, Vestnik USATU, 9, 125-135 (2007)

[6] Ouhadan, A.; Elkinani, E. H., Exact solutions of time fractional Kolmogorov equation by using lie symmetry analysis, J Fract Calc Appl, 5, 97-104 (2014)

[7] Wang, G.w.; Liu, X.i.; Zhang, Y., Lie symmetry analysis to the time fractional generalized fifth-order KDV equation, Commun Nonlinear Sci Numer Simul, 18, 2321-2326 (2013)

[8] Wang, G. W.; Xu, T. Z.; Feng, T., Lie symmetry analysis and explicit solutions of the time fractional fifth-order KDV equation, PLoS ONE, 9, e88336 (2014)

[9] Djordjevic, V. D.; Atanackovic, T. M., Similarity solutions to nonlinear heat conduction and Burgers/Korteweg-deVries fractional equations, J. Comput. Appl. Math., 222, 701-714 (2008)

[10] Singla, K.; Gupta, R. K., Space-time fractional nonlinear partial differential equations: symmetry analysis and conservation laws, Nonlinear Dyn, 89, 321-331 (2017)

[11] Riewe, F., Mechanics with fractional derivatives, Phys Rev E, 55, 3581-3592 (1997)

[12] Riewe, F., Nonconservative lagrangian and hamiltonian mechanics, Phys Rev E, 53, 1890-1899 (1996)

[13] Agrawal, O. P., Fractional variational calculus and the transversality conditions, J Phys A Math Gen, 39, 10375-10384 (2006)

[14] Agrawal, O. P., Fractional variational calculus in terms of Riesz fractional derivatives, J Phys A Math Gen, 40, 6287-6303 (2007)

[15] Tarasov, V. E.; Zaslavsky, G. M., Conservation laws and Hamilton's equations for systems with long-range interaction and memory, Commun Nonlinear Sci Numer Simul, 13, 1860-1878 (2008)

[16] Buckwar, E.; Yu, L., Invariance of a partial differential equation of fractional order under the lie group of scaling transforma-

tions, J Math Anal Appl, 227, 81-97 (1998)

[17] Hu, X.; Zhang, L., A compact finite difference scheme for the fourth-order fractional diffusion-wave system, Comput Phys Commun, 182, 1645-1650 (2011)

[18] Mao, Z.; Xiao, A.; Yu, Z.; Shi, L., Finite difference and sin-collocation approximations to a class of fractional diffusion-wave equations, J Appl Math, 11, 536030 (2014)

[19] Heydari, M. H.; Hooshmandasl, M. R.; Maalek Ghaini, F. M.; Cattani, C., Wavelets method for the time fractional diffusion-wave equation, Phys Lett A, 379, 71-76 (2015)

[20] Wang, G.; Xu, T., Symmetry properties and explicit solutions of the nonlinear time fractional KDV equation, Bound Value Probl, 1, 232 (2013)

[21] Kiryakova, V., Generalised fractional calculus and applications, Pitman research notes in mathematics Vol. 301, 388 (1994), Longman

[22] Podlubny, I., Fractional differential equations. an introduction to fractional derivatives, fractional differential equations, Some methods of their solution and some of their applications (1998), San Diego: San Diego CA: Academic

[23] Seybold, H. J.; Hilfer, R., Numerical results for the generalized Mittag-Leffler function, Fract Calc Appl Anal, 8, 127-139 (2005)

[24] Humbert, P., Quelques resultats relatifs a la fonction de Mittag-Leffler, CR Acad Sci Paris, 236, 1467-1468 (1953)

[25] Tarasov Vasily, E., On chain rule for fractional derivatives, Commun Nonlinear Sci Numer Simul, 30, 1-4 (2016)

[26] Dold, A.; Eckmann, B., Lecture notes in mathematics: fractional calculus and its applications, Proceeding of the international conference held at the university of New Haven (1975), Springer-Verlag. Berlin-Heidelberg: Springer-Verlag. Berlin-Heidelberg New York

[27] Diethelm, K., The analysis of fractional differential equations: an application-oriented exposition using differential operators of Caputo type (2004), Springer Heidelberg: Springer Heidelberg Dordrecht London. New York

[28] Stephani, H.; MacCallum, M., Differential equations: their solution using symmetries (1989), Cambridge University Press: Cambridge University Press Cambridge

[29] Gazizov, R. K.; Kasatkin, A. A.; Lukashchuk, S. Y., Group-invariant solutions of fractional differential equations, (Machado, J.; Luo, A.; Barbosa, R.; Silva, M.; Figueiredo, L., Nonlinear Science and Complexity (2011), Springer: Springer Dordrecht)

[30] Gazizov, R. K.; Kasatkin, A. A.; Lukashchuk, S. Y., Symmetry properties of fractional diffusion equations, Phys Scr, T136, 014016 (2009)

[31] Khorshidi, M.; Nadjafikhah, M.; Jafari, H., Fractional derivative generalization of Noether's theorem, Open Math, 13, 940-947 (2015)

[32] Zhou, F.; Xu, X., Numerical solution of the convection diffusion equations by the second kind Chebyshev wavelets, Appl Math Comput, 247, 353-367 (2014)

[33] Abd-Elhameed, W. M.; Doha, E. H.; Youssri, Y. H., New spectral second kind Chebyshev wavelets algorithm for solving linear and nonlinear second-order differential equations involving singular and Bratu type equations, Abstr Appl Anal, 9, 715756 (2013)

[34] Doha, E. H.; Abd-Elhameed, W. M.; Youssri, Y. H., Second kind Chebyshev operational matrix algorithm for solving differential equations of Lane-Emden type, New Astron, 23-24, 113-117 (2013)