Diekmann, O.; **Heesterbeek, J. A. P.**; **Metz, J. A. J.**

On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations. (English) Zbl 0726.92018

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Let a population of individuals be described by a variable ξ (h- state) and let $S = S(\xi)$ be the density function of susceptibles describing the steady demographic state in the absence of the disease. Moreover, define the following operator:

$$(1) \quad K((S)\phi)(\xi) = S(\xi) \int_{\Omega} \int_{0}^{\infty} A(\tau, \xi, \eta) d\tau \phi(\eta) d\eta.$$

In (1) Ω denotes the h-state space, A(τ,ξ,η) is the expected infectivity of an individual which was infected τ units of time ago with h-state η towards a susceptible which has h-state ξ ; finally ϕ is the density describing the distribution of the individuals. K(S) is called the next- generation operator. K(S) is a positive operator and its spectral radius r(K(S)) is the dominant eigenvalue ρ_d of K(S). Thus, the basic reproduction ratio R_0 which states the threshold criterion to have disease invasion or not, is defined as $R_0 = \rho_d$.

The computation of R_0 and a threshold criterion are given in the case when K(S) is of one-dimensional range. Biologically, this corresponds to the situation in which the distribution of the ones who become infected is independent of the state of the one who transmits the infection. In this case, we speak of a separable mixing rate. Also generalizations of cases when K(S) has finite-dimensional range are given. Threshold criteria for R_0 can also be derived in the cases when individuals preferentially mix with their own kind and otherwise practise weighted homogeneous mixing. Examples are given for discrete h- states and sexually transmitted diseases. There is also a study of the above cases in age-dependent models.

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