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Inverse spectral problem for the density of a vibrating elastic membrane. (English)

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Summary: This paper is concerned with the recovery of an unknown symmetric density function in the weighted Helmholtz equation with Dirichlet boundary conditions from the lowest few eigenvalues. By using the piecewise constant function to approximate the density function and using the Rayleigh-Ritz approach to discretize the differential equation, the continuous inverse eigenvalue problem is converted to a related matrix inverse eigenvalue problem and then a least squares problem for the discrete model is formulated. The solution of the least squares problem via an iterative method is discussed and then an approximation to the unknown density is recovered. Numerical experiments are given to confirm its competitiveness.

MSC:

74K15 Membranes
74G75 Inverse problems in equilibrium solid mechanics
74H45 Vibrations in dynamical problems in solid mechanics
35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
35R30 Inverse problems for PDEs

Keywords:

Helmholtz equation; inverse eigenvalue problem; density function; piecewise constant; iterative method

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