Javili, Ali

A note on traction continuity across an interface in a geometrically non-linear framework. (English) Zbl 07254365

Math. Mech. Solids 24, No. 8, 2478-2496 (2019).

Summary: The objective of this contribution is to elaborate on the notion of "traction continuity" across an interface at finite deformations. The term interface corresponds to a zero-thickness model representing the interphase between different constituents in a material. Commonly accepted interface models are the cohesive interface model and the elastic interface model. Both the cohesive and elastic interface models are the limit cases of a generalized interface model. This contribution aims to rigorously analyze the concept of the traction jump for the general interface model. The governing equations of the general interface model in the material as well as spatial configurations are derived and the traction jump across the interface for each configuration is highlighted. It is clearly shown that the elastic interface model undergoes a traction jump in both the material and spatial configurations according to a generalized Young-Laplace equation. For the cohesive interface model, however, while the traction field remains continuous in the material configuration, it can suffer a jump in the spatial configuration. This finding is particularly important since the cohesive interface model is based on the assumption of traction continuity across the interface and that the term "traction" often refers to the spatial configuration and not the material one. Thus, additional care should be taken when formulating an interface model in a geometrically non-linear framework. The theoretical findings for various interface models are carefully illustrated via a series of two-dimensional and three-dimensional numerical examples using the finite element method.

Reviewer: Reviewer (Berlin)

MSC:

74 Mechanics of deformable solids

Keywords:

general interface model; traction jump; interface elasticity; cohesive interface

Full Text: DOI

References:

[1] Matous, K, Kulkarni, MG, Geubelle, PH. Multiscale cohesive failure modeling of heterogeneous adhesives. J Mech Phys Solids 2008; 56(4): 1511-1533. · Zbl 1171.74433

[2] McBride, A, Mergheim, J, Javili, A et al. Micro-to-macro transitions for heterogeneous material layers accounting for in-plane stretch. J Mech Phys Solids 2012; 60(6): 1221-1239.

[3] Fried, E, Grach, G. An order-parameter-based theory as a regularization of a sharp-interface theory for solid-solid phase transitions. Arch Rat Mech Anal 1997; 138(4): 355-404. · Zbl 0910.73006

[4] Petryk, H, Stupkiewicz, S. Interfacial energy and dissipation in martensitic phase transformations. Part I: Theory. J Mech Phys Solids 2010; 58(3): 390-408. · Zbl 1193.74046

[5] Petryk, H, Stupkiewicz, S, Maciejewski, G. Interfacial energy and dissipation in martensitic phase transformations. Part II: Size effects in pseudoelasticity. J Mech Phys Solids 2010; 58(3): 373-389. · Zbl 1193.74047

[6] Levitas, VI, Warren, JA. Phase field approach with anisotropic interface energy and interface stresses: Large strain formulation. J Mech Phys Solids 2016; 91: 94-125.

[7] Tuma, K, Stupkiewicz, S, Petryk, H. Size effects in martensitic microstructures: Finite-strain phase field model versus sharp-interface approach. J Mech Phys Solids 2016; 95: 284-307.

[8] Bövik, P. On the modelling of thin interface layers in elastic and acoustic scattering problems. Q J Mech Appl Math 1994; 47(1): 17-42. · Zbl 0803.73025

[9] Benveniste, Y, Miloh, T. Imperfect soft and stiff interfaces in two-dimensional elasticity. Mech Mater 2001; 33(6): 309-323.

[10] Benveniste, Y. A general interface model for a three-dimensional curved thin anisotropic interphase between two anisotropic media. J Mech Phys Solids 2006; 54(4): 708-734. · Zbl 1120.74323

[11] Monchiet, V, Bonnet, G. Interfacial models in viscoplastic composites materials. Int J Eng Sci 2010; 48(12): 1762-1768. · Zbl

1231.74082

[12] Benveniste, Y, Milton, GW. The effective medium and the average field approximations vis-a-vis the Hashin-Shtrikman bounds. I. The self-consistent scheme in matrix-based composites. J Mech Phys Solids 2010; 58(7): 1026-1038. · Zbl 1244.74009

[13] Benveniste, Y, Milton, GW. The effective medium and the average field approximations vis-a-vis the Hashin-Shtrikman bounds. II. The generalized self-consistent scheme in matrix-based composites. J Mech Phys Solids 2010; 58(7): 1039-1056. · Zbl 1244.74010

[14] Gu, ST, Monteiro, E, He, QC. Coordinate-free derivation and weak formulation of a general imperfect interface model for thermal conduction in composites. Composites Sci Technol 2011; 71(9): 1209-1216.

[15] Gu, ST, He, QC. Interfacial discontinuity relations for coupled multifield phenomena and their application to the modeling of thin interphases as imperfect interfaces. J Mech Phys Solids 2011; 59(7): 1413-1426. · Zbl 1270.74037

[16] Pavanello, F, Manca, F, Luca Palla, P et al. Generalized interface models for transport phenomena: Unusual scale effects in composite nanomaterials. J Appl Phys 2012; 112: 1-10.

[17] Gu, ST, Liu, JT, He, QC. Size-dependent effective elastic moduli of particulate composites with interfacial displacement and traction discontinuities. International Journal of Solids and Structures 2014; 51(13): 2283-2296.

[18] Chen, T, Chiu, MS, Weng, CN. Derivation of the generalized Young-Laplace equation of curved interfaces in nanoscaled solids. J Appl Phys 2006; 100(7): 074308.

[19] Javili, A, Mcbride, A, Steinmann, P. Thermomechanics of solids with lower-dimensional energetics: On the importance of surface, interface, and curve structures at the nanoscale. A unifying review. Appl Mech Rev 2013; 65(1): 010802.

[20] Moeckel, GP. Thermodynamics of an interface. Arch Rat Mech Anal 1975; 57(3): 255-280. · Zbl 0338.73001

[21] Murdoch, AI. A thermodynamical theory of elastic material interfaces. Q J Mech Appl Math 1976; 29(3): 245-275. · Zbl 0398.73003

[22] Daher, N, Maugin, GA. The method of virtual power in continuum mechanics application to media presenting singular surfaces and interfaces. Acta Mechanica 1986; 60(3-4): 217-240. · Zbl 0594.73004

[23] Dell'Isola, F, Romano, A. On the derivation of thermomechanical balance equations for continuous systems with a nonmaterial interface. Int J Eng Sci 1987; 25(11-12): 1459-1468. · Zbl 0624.73001

[24] Fried, E, Gurtin, ME. Thermomechanics of the interface between a body and its environment. Continuum Mech Thermodyn 2007; 19(5): 253-271. · Zbl 1160.74303

[25] Javili, A, Ottosen, NS, Ristinmaa, M et al. Aspects of interface elasticity theory. Math Mech Solids 2017; DOI: 10.1177/1081286517699041. · Zbl 1401.74042

[26] Gurtin, ME, Murdoch, AI. A continuum theory of elastic material surfaces. Arch Rat Mech Anal 1975; 57(4): 291-323. · Zbl 0326.73001

[27] Gurtin, ME, Weissmüller, J, Larche, F. A general theory of curved deformable interfaces in solids at equilibrium. Philos Mag A 1998; 78(5): 1093-1109.

[28] Steigmann, DJ, Ogden, RW. Elastic surface-substrate interactions. Proc R Soc A Math Phys Eng Sci 1999; 455(1982): 437-474. · Zbl 0926.74016

[29] Fried, E, Todres, RE. Mind the gap: The shape of the free surface of a rubber-like material in proximity to a rigid contactor. J Elasticity 2005; 80(1-3): 97-151. · Zbl 1197.74013

[30] Huang, ZP, Wang, J. A theory of hyperelasticity of multi-phase media with surface/interface energy effect. Acta Mechanica 2006; 182(3-4): 195-210. · Zbl 1121.74007

[31] Steinmann, P. On boundary potential energies in deformational and configurational mechanics. J Mech Phys Solids 2008; 56(3): 772-800. · Zbl 1149.74006

[32] Dingreville, R, Qu, J. Interfacial excess energy, excess stress and excess strain in elastic solids: Planar interfaces. J Mech Phys Solids 2008; 56(5): 1944-1954. · Zbl 1162.74316

[33] Duan, HL, Wang, J, Karihaloo, BL. Theory of Elasticity at the Nanoscale. Adv Appl Mech 2009; 42: 1-68.

[34] Wang, Y, Weissmüller, J, Duan, HL. Mechanics of corrugated surfaces. J Mech Phys Solids 2010; 58(10): 1552-1566. · Zbl 1200.74008

[35] Wang, ZQ, Zhao, YP, Huang, ZP. The effects of surface tension on the elastic properties of nano structures. Int J Eng Sci 2010; 48(2): 140-150.

[36] Altenbach, H, Eremeyev, VA. On the shell theory on the nanoscale with surface stresses. Int J Eng Sci 2011; 49(12): 1294-1301. · Zbl 1423.74561

[37] Chhapadia, P, Mohammadi, P, Sharma, P. Curvature-dependent surface energy and implications for nanostructures. J Mech Phys Solids 2011; 59(10): 2103-2115. · Zbl 1270.74018

[38] Zemlyanova, AY. The effect of a curvature-dependent surface tension on the singularities at the tips of a straight interface crack. Q J Mech Appl Math 2013; 66(2): 199-219. · Zbl 1291.74165

[39] Dingreville, R, Hallil, A, Berbenni, S. From coherent to incoherent mismatched interfaces: A generalized continuum formulation of surface stresses. J Mech Phys Solids 2014; 72(1): 40-60. · Zbl 1328.74012

[40] Gao, X, Huang, Z, Qu, J et al. A curvature-dependent interfacial energy-based interface stress theory and its applications to nano-structured materials: (I) General theory. J Mech Phys Solids 2014; 66(1): 59-77. · Zbl 1323.74008

[41] Cordero, NM, Forest, S, Busso, EP. Second strain gradient elasticity of nano-objects. J Mech Phys Solids 2016; 97: 92-124.

[42] Liu, L, Yu, M, Lin, H et al. Deformation and relaxation of an incompressible viscoelastic body with surface viscoelasticity. J Mech Phys Solids 2017; 98: 309-329.

[43] Barenblatt, GI. The formation of equilibrium cracks during brittle fracture. General ideas and hypotheses. Axially-symmetric cracks. J Appl Math Mech 1959; 23(3): 622-636. · Zbl 0095.39202

[44] Barenblatt, GI. The mathematical theory of equilibrium cracks in brittle fracture. Adv Appl Mech 1962; 7: 55-129.

[45] Dugdale, D. Yielding of steel sheets containing slits. J Mech Phys Solids 1960; 8(2): 100-104.

[46] Needleman, A. A continuum model for void nucleation by inclusion debonding. J Appl Mech 1987; 54: 525-531. · Zbl 0626.73010

[47] Xu, XP, Needleman, A. Numerical simulations of fast crack growth in brittle solids. J Mech Phys Solids 1994; 42(9): 1397-1434. · Zbl 0825.73579

[48] Ortiz, M, Pandolfi, A. Finite-deformation irreversible cohesive elements for three-dimensional crack-propagation analysis. Int J Numer Meth Eng 1999; 44(9): 1267-1282. · Zbl 0932.74067

[49] Tijssens, MG, Sluys, BL, Van der Giessen, E. Numerical simulation of quasi-brittle fracture using damaging cohesive surfaces. Eur J Mech A Solids 2000; 19(5): 761-779. · Zbl 0993.74073

[50] Alfano, G, Crisfield, MA. Finite element interface models for the delamination analysis of laminated composites: mechanical and computational issues. Int J Numer Meth Eng 2001; 50: 1701-1736. · Zbl 1011.74066

[51] Gasser, TC, Holzapfel, GA. Geometrically non-linear and consistently linearized embedded strong discontinuity models for 3D problems with an application to the dissection analysis of soft biological tissues. Comput Meth Appl Mech Eng 2003; 192(47-48): 5059-5098. · Zbl 1088.74541

[52] van den Bosch, MJ, Schreurs, PJG, Geers, MGD. An improved description of the exponential Xu and Needleman cohesive zone law for mixed-mode decohesion. Eng Fract Mech 2006; 73(9): 1220-1234.

[53] Fagerström, M, Larsson, R. Theory and numerics for finite deformation fracture modelling using strong discontinuities. Int J Numer Meth Eng 2006; 66(6): 911-948. · Zbl 1110.74815

[54] Charlotte, M, Laverne, J, Marigo, JJ. Initiation of cracks with cohesive force models: a variational approach. Eur J Mech A Solids 2006; 25(4): 649-669. · Zbl 1187.74159

[55] Park, K, Paulino, GH, Roesler, JR. A unified potential-based cohesive model of mixed-mode fracture. J Mech Phys Solids 2009; 57(6): 891-908.

[56] Park, K, Paulino, GH. Cohesive zone models: a critical review of traction-separation relationships across fracture surfaces. Appl Mech Rev 2013; 64(6): 060802.

[57] Dimitri, R, Trullo, M, De Lorenzis, L et al. Coupled cohesive zone models for mixed-mode fracture: A comparative study. Eng Fract Mech 2015; 148: 145-179.

[58] Wu, C, Gowrishankar, S, Huang, R et al. On determining mixed-mode traction-separation relations for interfaces. Int J Fract 2016; 202(1): 1-19.

[59] Qian, J, Lin, J, Xu, GK et al. Thermally assisted peeling of an elastic strip in adhesion with a substrate via molecular bonds. J Mech Phys Solids 2017; 101: 197-208.

[60] Marsden, JE, Hughes, TJR. Mathematical Foundations of Elasticity. New York: Dover, 1994.

[61] Holzapfel, GA. Nonlinear Solid Mechanics: A Continuum Approach for Engineering. Hoboken, NJ: John Wiley \& Sons, 2000. · Zbl 0980.74001

[62] Gurtin, ME, Fried, E, Anand, L. The Mechanics and Thermodynamics of Continua. Cambridge: Cambridge University Press, 2009.

[63] Bowen, RM, Wang, CC. Introduction to Vectors and Tensors: Linear and Multilinear Algebra. New York: Plenum Press, 1976. · Zbl 0329.53008

[64] Kreyszig, E. Differential Geometry. New York: Dover, 1991.

[65] Ciarlet, PG. An Introduction to Differential Geometry with Applications to Elasticity. New York: Springer, 2005. · Zbl 1100.53004

[66] Steinmann, P. Geometrical Foundations of Continuum Mechanics: An Application to First- and Second-Order Elasticity and Elasto-Plasticity. New York: Springer, 2015. · Zbl 1329.74003

[67] Javili, A. Variational formulation of generalized interfaces for finite deformation elasticity. Math Mech Solids 2017; DOI: 10.1177/1081286517719938. · Zbl 1425.74199

[68] Javili, A, Steinmann, P, Mosler, J. Micro-to-macro transition accounting for general imperfect interfaces. Comput Meth Appl Mech Eng 2017; 317: 274-317. · Zbl 1439.74333

[69] Javili, A, Dell'Isola, F, Steinmann, P. Geometrically nonlinear higher-gradient elasticity with energetic boundaries. J Mech Phys Solids 2013; 61(12): 2381-2401. · Zbl 1294.74014

[70] Vossen, BG, Schreurs, PJG, van der Sluis, O et al. On the lack of rotational equilibrium in cohesive zone elements. Comput Meth Appl Mech Eng 2013; 254: 146-153. · Zbl 1297.74110

[71] Ogden, R. Large deformation isotropic elasticity - on the correlation of theory and experiment for incompressible rubberlike solids. Proc R Soc A 1972; 326(1567): 565-584. · Zbl 0257.73034

[72] Linder, C, Tkachuk, M, Miehe, C. A micromechanically motivated diffusion-based transient network model and its incorporation into finite rubber viscoelasticity. J Mech Phys Solids 2011; 59(10): 2134-2156. · Zbl 1270.74035