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Subcritical phase of d-dimensional Poisson-Boolean percolation and its vacant set. (Phase sous-critique du modèle de percolation Poisson-Booléen et de son complémentaire en dimension d.) (English. French summary) Zbl 07249463

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For every $r > 0$, define the two functions of λ as $\theta_r(\lambda) := \mathbf{P}_\lambda[0 \leftrightarrow \partial B_r]$ and $\theta(\lambda) := \lim_{r\to\infty} \theta_r(\lambda)$. Define the critical parameter $\lambda_c = \lambda_c(d)$ of the model by the formula $\lambda_c := \inf\{\lambda \geq 0 : \theta(\lambda) > 0\}$. The authors introduce the another critical parameter to discuss Poisson-Boolean percolation as $\widetilde{\lambda}_c := \inf\left\{\lambda \geq 0 : \inf_{r>0} \mathbb{P}_\lambda[B_\lambda \leftrightarrow \partial B_{2r}] > 0\right\}$. The authors prove the following main result: "Theorem 1.2 (Sharpness for Poisson-Boolean percolation). – Fix $d \geq 2$ and assume that $\int_{\mathbb{R}_+} r^{5d-3} d\mu(r) < \infty$. Then, we have that $\lambda_c = \widetilde{\lambda}_c$. Furthermore, there exists $c > 0$ such that $\theta(\lambda) > c(\lambda - \lambda_c)$ for any $\lambda \geq \lambda_c$."

The authors give a brief description of the general strategy to prove the main theorem. Three properties of the Poisson-Boolean percolation are introduced. Then, the authors present some new results concerning the behavior of Poisson-Boolean percolation when $\lambda < \widetilde{\lambda}_c$. If there exists $c > 0$ such that $\mu[r, \infty] \leq \exp(-cr)$ for every $r \geq 1$, then, for every $\lambda < \widetilde{\lambda}_c$, the authors prove that there exists $c_\lambda > 0$ such that for every $r > 1$, $\theta_r(\lambda) \leq \exp(-c_\lambda r)$.

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MSC:

82B43 Percolation
82B26 Phase transitions (general) in equilibrium statistical mechanics
82B27 Critical phenomena in equilibrium statistical mechanics

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