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Complex dynamics of a discrete-time prey-predator system with Leslie type: stability, bifurcation analyses and chaos. (English) Zbl 1453.39014

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Applying the forward Euler method to the continuous-time prey-predator model with Leslie type functional response,

$$\begin{cases} \frac{dN(t)}{dt} &= r_1 N(t) - \epsilon P(t) N(t), \\ \frac{dP(t)}{dt} &= P(t)\left(r_2 - \theta \frac{P(t)}{N(t)}\right), \end{cases}$$

the authors obtain the following discrete-time prey-predator system,

$$\begin{cases} N_{t+1} &= N_t + \delta N_t (r_1 - \epsilon P_t), \\ P_{t+1} &= P_t + \delta P_t \left(r_2 - \theta \frac{P_{t+1}}{N_{t+1}}\right), \end{cases}$$

where N and P represent prey and predator, respectively. Here δ is the integral step size. The discrete system has only a single positive equilibrium $(\overline{N}, \overline{P}) = (\frac{\theta r_1}{\epsilon r_2}, \frac{r_1}{\epsilon})$. First, through linearization, conditions on the local stability of $(\overline{N}, \overline{P})$ are obtained. Then, choosing δ as the bifurcation parameter, the flip bifurcation and the Neimark-Sacker bifurcation arising from $(\overline{N}, \overline{P})$ are analyzed by employing the center manifold theorem and normal form theory. These theoretical results are not only supported but also extended by numerical simulations. For example, large values of δ can lead to chaotic behavior, which is impossible for the continuous-time counterpart.

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MSC:

39A60 Applications of difference equations
65L05 Numerical methods for initial value problems
65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
65P20 Numerical chaos
37N25 Dynamical systems in biology
39A30 Stability theory for difference equations
39A28 Bifurcation theory for difference equations
92D25 Population dynamics (general)

Keywords:

discrete prey-predator system; stability; flip bifurcation; Neimark-Sacker bifurcation; chaotic behavior

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