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Testing for the significance of functional covariates. (English) Zbl 1448.62207

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Summary: We consider the problem of testing for the nullity of conditional expectations of Hilbert space-valued random variables. We allow for conditioning variables taking values in finite or infinite Hilbert spaces. This testing problem occurs, for instance, when checking the goodness-of-fit or the effect of some infinite-dimensional covariates in regression models for functional data. Testing the independence, between a finite dimensional variable and a functional one, is another example that could be treated in our framework. We propose a new test based on kernel smoothing. The test statistic is asymptotically standard normal under the null hypothesis provided the smoothing parameter tends to zero at a suitable rate. The one-sided test is consistent against any fixed alternative, as well as against local alternatives à la Pitman and uniformly against classes of regular alternatives approaching the null hypothesis. In particular, we show that neither the dimension of the outcome nor the dimension of the functional covariates influences the theoretical power of the test against such alternatives. Simulation experiments and a real data application using a variable-domain functional regression model illustrate the performance of the new test.

MSC:

62R10 Functional data analysis
62G10 Nonparametric hypothesis testing
62G20 Asymptotic properties of nonparametric inference

Keywords:

functional data; goodness-of-fit test; kernel smoothing; U-statistics

Software:

fda (R)

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