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On geometric posets and partial matroids. (English) Zbl 07232921

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Summary: The aim of this paper is to extend the notions of geometric lattices, semimodularity and matroids in the framework of finite posets and related systems of sets. We define a geometric poset as one which is atomistic and which satisfies particular conditions connecting elements to atoms. Next, by using a suitable partial closure operator and the corresponding partial closure system, we define a partial matroid. We prove that the range of a partial matroid is a geometric poset under inclusion, and conversely, that every finite geometric poset is isomorphic to the range of a particular partial matroid. Finally, by introducing a new generalization of semimodularity from lattices to posets, we prove that a poset is geometric if and only if it is atomistic and semimodular.

MSC:

06A15 Galois correspondences, closure operators (in relation to ordered sets)

06A06 Partial orders, general

Keywords:

partial closure operator; partial closure system; centralized system; geometric posets; semimodularity

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