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A meshless generalized finite difference method for 2D elasticity problems. (English)

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Summary: In this paper, a meshless generalized finite difference (FD) method is developed and presented for solving 2D elasticity problems. Different with other types of generalized FD method (GFDM) commonly constructed with moving least square (MLS) or radial basis function (RBF) shape functions, the present method is developed based upon B-spline based shape function. The method is a truly meshless approach. Key aspects attributed to the method are: B-spline basis functions augmented with polynomials are employed to construct its shape function. This allows B-splines with lower order to be chosen for the approximation and keeping the efficiency of computation related to tensor product operation of B-spline basis functions. In addition, as distribution of stencil nodes affects numerical performance of generalized FD method, neighboring nodes from triangle cells surrounding a center node are selected for building the supporting domains. While meeting compact stencil requirement, the selection eliminates necessity for determining appropriate number of supporting nodes or size of supporting domains. As a result, the proposed method shows good numerical approximation and accuracy for 2D elasticity problems. Numerical examples are presented to show the effectiveness of the proposed method for solving several 2D elasticity problems in various geometries.

MSC:

74S20 Finite difference methods applied to problems in solid mechanics
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

Keywords:

meshless; B-spline shape function; generalized FD; triangle cells; 2D elasticity problems

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