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**Optimal dividend and capital injection strategy with a penalty payment at ruin: restricted dividend payments.** (English) [Zbl 1445.91055](#)

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Summary: In this paper, we study the optimal dividend and capital injection problem with the penalty payment at ruin. The dividend strategy is assumed to be restricted to a small class of absolutely continuous strategies with bounded dividend density. By considering the surplus process killed at the time of ruin, we transform the problem to a combined stochastic and impulse control one up to ruin with a free boundary at zero. We illustrate the theoretical verifications for different types of capital injection strategies comparing to the conventional results given in the literature, where the capital injections are made before the time of ruin. Under the assumption of restricted dividend density, the value function is proved as the unique increasing, bounded, Lipschitz continuous and upper semi-continuous at zero viscosity solution to the corresponding quasi-variational Hamilton-Jacobi-Bellman (HJB) equation. The uniqueness of such class of viscosity solutions is shown by considering its boundary condition at infinity. The optimality of a specific band-type strategy is proved for the case when the premium rate is (i) greater than or (ii) less than the ceiling dividend rate respectively. Some numerical examples are presented under the exponential and gamma claim size assumptions.

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**MSC:**

[91G05](#) Actuarial mathematics

[49L25](#) Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games

**Keywords:**

[dividend and capital injection](#); [penalty payment at ruin](#); [quasi-variational HJB equation](#); [viscosity solution](#); [band strategy](#)

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