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Additive-quadratic functional inequalities. (English) [Zbl 07225659](#)

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Summary: In this paper, we introduce and solve the following additive-quadratic s -functional inequalities:

$$\begin{aligned} & \|f(x+y) + f(x-y) - 2f(x) - f(y) - f(-y)\| \\ & \leq \left\| s \left(2f\left(\frac{x+y}{2}\right) - f(x) - f(y) + f\left(\frac{x-y}{2}\right) + f\left(\frac{y-x}{2}\right) \right) \right\|, \end{aligned} \quad (1)$$

where s is a fixed nonzero complex number with $|s| < \sqrt{2}$, and

$$\begin{aligned} & \left\| 2f\left(\frac{x+y}{2}\right) - f(x) - f(y) + f\left(\frac{x-y}{2}\right) + f\left(\frac{y-x}{2}\right) \right\| \\ & \leq \|s(f(x+y) + f(x-y) - 2f(x) - f(y) - f(-y))\|, \end{aligned} \quad (2)$$

where s is a fixed nonzero complex number with $|s| < \frac{1}{2}$. Using the direct method and the fixed point method, we prove the Hyers-Ulam stability of the additive-quadratic s -functional inequalities (1) and (2) in complex Banach spaces.

For the entire collection see [\[Zbl 1446.65002\]](#).

MSC:

65Jxx Numerical analysis in abstract spaces

49Jxx Existence theories in calculus of variations and optimal control

Full Text: [DOI](#)

References:

- [1] T. Aoki, On the stability of the linear transformation in Banach spaces. *J. Math. Soc. Japan* 2, 64-66 (1950) · [Zbl 0040.35501](#)
- [2] L. Cădariu, V. Radu, Fixed points and the stability of Jensen's functional equation. *J. Inequal. Pure Appl. Math.* 4(1), 4 (2003) · [Zbl 1043.39010](#)
- [3] L. Cădariu, V. Radu, On the stability of the Cauchy functional equation: a fixed point approach. *Grazer Math. Ber.* 346, 43-52 (2004) · [Zbl 1060.39028](#)
- [4] L. Cădariu, V. Radu, Fixed point methods for the generalized stability of functional equations in a single variable. *Fixed Point Theory Appl.* 2008, 749392 (2008) · [Zbl 1146.39040](#)
- [5] L. Cădariu, L. Găvruta, P. Găvruta, On the stability of an affine functional equation. *J. Nonlinear Sci. Appl.* 6, 60-67 (2013) · [Zbl 1296.39024](#)
- [6] A. Chahbi, N. Bounader, On the generalized stability of d'Alembert functional equation. *J. Nonlinear Sci. Appl.* 6, 198-204 (2013) · [Zbl 1296.39025](#)
- [7] P.W. Cholewa, Remarks on the stability of functional equations. *Aequationes Math.* 27, 76-86 (1984) · [Zbl 0549.39006](#)
- [8] J. Diaz, B. Margolis, A fixed point theorem of the alternative for contractions on a generalized complete metric space. *Bull. Am. Math. Soc.* 74, 305-309 (1968) · [Zbl 0157.29904](#)
- [9] N. Eghbali, J.M. Rassias, M. Taheri, On the stability of a k -cubic functional equation in intuitionistic fuzzy n -normed spaces. *Results Math.* 70, 233-248 (2016) · [Zbl 1360.39022](#)
- [10] G.Z. Eskandani, P. Găvruta, Hyers-Ulam-Rassias stability of p -exiderized Cauchy functional equation in 2-Banach spaces. *J. Nonlinear Sci. Appl.* 5, 459-465 (2012) · [Zbl 1297.39030](#)
- [11] P. Găvruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings. *J. Math. Anal. Appl.* 184, 431-436 (1994) · [Zbl 0818.46043](#)
- [12] D.H. Hyers, On the stability of the linear functional equation. *Proc. Natl. Acad. Sci. U.S.A.* 27, 222-224 (1941) · [Zbl 67.0424.01](#)
- [13] G. Isac, T.M. Rassias, Stability of ψ -additive mappings: applications to nonlinear analysis. *Int. J. Math. Math. Sci.* 19, 219-228

(1996) · [Zbl 0843.47036](#)

- [14] H. Khodaei, On the stability of additive, quadratic, cubic and quartic set-valued functional equations. *Results Math.* 68, 1-10 (2015) · [Zbl 1330.39029](#)
- [15] D. Miheţ, V. Radu, On the stability of the additive Cauchy functional equation in random normed spaces. *J. Math. Anal. Appl.* 343, 567-572 (2008) · [Zbl 1139.39040](#)
- [16] C. Park, Additive ρ -functional inequalities and equations. *J. Math. Inequal.* 9, 17-26 (2015) · [Zbl 1314.39026](#)
- [17] C. Park, Additive ρ -functional inequalities in non-Archimedean normed spaces. *J. Math. Inequal.* 9, 397-407 (2015) · [Zbl 1323.39023](#)
- [18] V. Radu, The fixed point alternative and the stability of functional equations. *Fixed Point Theory* 4, 91-96 (2003) · [Zbl 1051.39031](#)
- [19] T.M. Rassias, On the stability of the linear mapping in Banach spaces. *Proc. Am. Math. Soc.* 72, 297-300 (1978) · [Zbl 0398.47040](#)
- [20] F. Skof, Propriet locali e approssimazione di operatori. *Rend. Sem. Mat. Fis. Milano* 53, 113-129 (1983) · [Zbl 0599.39007](#)
- [21] S.M. Ulam, *A Collection of the Mathematical Problems* (Interscience Publ. New York, 1960) · [Zbl 0086.24101](#)

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