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Additive (ρ_1, ρ_2)-functional inequalities in complex Banach spaces. (English) Zbl 07225654
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Summary: In this paper, we introduce and solve the following additive (ρ_1, ρ_2)-functional inequalities:

$$\begin{aligned}\|f(x-y)-f(x)+f(y)\| \geq &\|\rho_1(f(x+y)-f(x)-f(y))\| \\ &+\|\rho_2(f(y-x)-f(y)+f(x))\|,\end{aligned} \tag{1}$$

where ρ_1 and ρ_2 are fixed complex numbers with $|\rho_1|+|\rho_2|>1$, and

$$\begin{aligned}\|f(x+y)-f(x)-f(y)\| \geq &\|\rho_1(f(x-y)-f(x)+f(y))\| \\ &+\|\rho_2(f(y-x)-f(y)+f(x))\|,\end{aligned} \tag{2}$$

where ρ_1 and ρ_2 are fixed complex numbers with $1+|\rho_1|>|\rho_2|>1$. Using the fixed point method and the direct method, we prove the Hyers-Ulam stability of the additive (ρ_1, ρ_2)-functional inequalities (2) and (1) in complex Banach spaces.

For the entire collection see [Zbl 1446.65002].

MSC:

65Jxx Numerical analysis in abstract spaces
49Jxx Existence theories in calculus of variations and optimal control

Full Text: DOI

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