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**Injective envelopes of transition systems and Ferrers languages.** (English) Zbl 1481.06022  
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Summary: We consider reflexive and involutive transition systems over an ordered alphabet  $A$  equipped with an involution. We give a description of the injective envelope of any two-element set in terms of Galois lattice, from which we derive a test of its finiteness. Our description leads to the notion of Ferrers language.

**MSC:**

- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 06D20 Heyting algebras (lattice-theoretic aspects)
- 54E35 Metric spaces, metrizable
- 46B85 Embeddings of discrete metric spaces into Banach spaces; applications in topology and computer science
- 68Q70 Algebraic theory of languages and automata

**Keywords:**

metric spaces; injective envelopes; transition systems; Ferrers languages; ordered sets; interval orders; well-quasi-order

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